

①

(b) Gauss-Seidel method :-

Let

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

be a system of linear equations.

We can find the values of x_1, x_2 and x_3 by using the following formulas :-

$$x_1^{(k+1)} = [b_1 - (a_{12}x_2^{(k)} + a_{13}x_3^{(k)})] / a_{11}$$

$$x_2^{(k+1)} = [b_2 - (a_{21}x_1^{(k+1)} + a_{23}x_3^{(k)})] / a_{22}$$

$$x_3^{(k+1)} = [b_3 - (a_{31}x_1^{(k+1)} + a_{32}x_2^{(k+1)})] / a_{33}$$

$$k = 0, 1, 2, \dots$$

Example :

By Gauss-Seidel method, solve the following system :

$$4x_1 + x_2 - 2x_3 = 1$$

$$x_1 + 3x_2 - x_3 = 8$$

$$x_1 - 7x_2 + 10x_3 = 2$$

where $x_1^{(0)} = 1, x_2^{(0)} = 3, x_3^{(0)} = 2$, two loops?

(2)

Solution

$$X_1^{(k+1)} = [b_1 - (a_{12}X_2^{(k)} + a_{13}X_3^{(k)})] / a_{11}$$

$$X_2^{(k+1)} = [b_2 - (a_{21}X_1^{(k+1)} + a_{23}X_3^{(k)})] / a_{22}$$

$$X_3^{(k+1)} = [b_3 - (a_{31}X_1^{(k+1)} + a_{32}X_2^{(k+1)})] / a_{33}$$

$$k = 0, 1, 2, \dots$$

$$X_1^{(1)} = [1 - (X_2^{(0)} - 2X_3^{(0)})] / 4 = [1 - (3 - 4)] / 4 = 0.5$$

$$X_2^{(1)} = [8 - (X_1^{(1)} - X_3^{(0)})] / 3 = [8 - (0.5 - 2)] / 3 = 3.167$$

$$X_3^{(1)} = [2 - (X_1^{(1)} - 7X_2^{(1)})] / 10 = [2 - (0.5 - 7(3.167))] / 10 = 2.367$$

$$X_1^{(2)} = [1 - (X_2^{(1)} - 2X_3^{(1)})] / 4 = [1 - (3.167 - 2(2.367))] / 4 = 0.643$$

$$X_2^{(2)} = [8 - (X_1^{(2)} - X_3^{(1)})] / 3 = [8 - (0.643 - 2.367)] / 3 = 3.241$$

$$X_3^{(2)} = [2 - (X_1^{(2)} - 7X_2^{(2)})] / 10 = [2 - (0.643 - 7(3.241))] / 10 = 2.403$$

(3)

Exercise:-

Solve the following system of linear equations, by Gauss-Seidel method:

$$-3X_1 + X_2 - X_3 = -1$$

$$3X_1 + 9X_2 + X_3 = 2$$

$$X_1 - 2X_2 + 4X_3 = 0$$

If $X_1^{(0)} = X_2^{(0)} = X_3^{(0)} = 0$, three loops?