Image Restoration

Image restoration attempts to reconstruct or recover an image that has been degraded by a degradation phenomenon. Thus, restoration techniques are oriented toward modeling the degradation and applying the inverse process in order to recover the original image. As in image enhancement, the ultimate goal of restoration techniques is to improve an image in some predefined sense.

Image restoration vs. image enhancement

	Image restoration	Image enhancement
1.	is an objective process	is a subjective process
2.	formulates a criterion of goodness that will yield an optimal estimate of the desired result	involves heuristic procedures designed to manipulate an image in order to satisfy the human visual system
3.	Techniques include noise removal and deblurring (removal of image blur)	Techniques include contrast stretching

Like enhancement techniques, restoration techniques can be performed in the spatial domain and frequency domain. For example, noise removal is applicable using spatial domain filters whereas deblurring is performed using frequency domain filters because image blur are difficult to approach in the spatial domain using small masks.

A Model of Image Degradation & Restoration

As shown in the next figure, image degradation is a process that acts on an input image *f(x,y)* through a degradation function *H* and an additive noise *η(x,y)*. It results in a degraded image *g(x,y)* such that:

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

where *h(x,y)* is the spatial representation of the degradation function and the symbol " * " indicates convolution.

Note that we only have the degraded image *g(x,y)*. The objective of restoration is to obtain an estimate $f^{\wedge}(x, y)$ of the original image. We want the estimate to be as close as possible to the original input image and, in general, the more we know about H and η, the closer $f^{\wedge}(x, y)$ will be to *f(x, y)*.

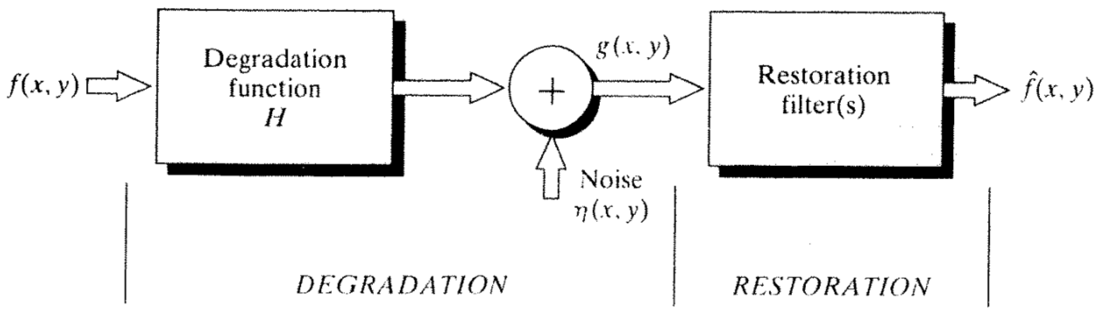


Figure 10.1 A model of the image degradation/restoration process

In the frequency domain, this model is equivalent to:

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

The approach that we will study is based on various types of image restoration filters. We assume that H is the identity operator, and we deal only with degradations due to noise.

Noise and its characteristics

Noise in digital images arises during:

- Acquisition: environmental conditions (light level & sensor temperature), and type of cameras
- and/or transmission – interference in the transmission channel

To remove noise we need to understand the spatial characteristics of noise and its frequency characteristics (Fourier spectrum).

Generally, spatial noise is assumed to be independent of position in an image and uncorrelated to the image itself (i.e. there is no correlation between pixel values and the values of noise components). Frequency properties refer to the frequency content of noise in the Fourier sense.

Noise Models

Spatial noise is described by the statistical behavior of the gray-level values in the noise component of the degraded image. Noise can be modeled as a random variable with a specific probability distribution function (PDF). Important examples of noise models include:

1. Gaussian Noise
2. Rayleigh Noise
3. Gamma Noise
4. Exponential Noise
5. Uniform Noise
6. Impulse (Salt & Pepper) Noise

Gaussian Noise

The PDF of Gaussian noise is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

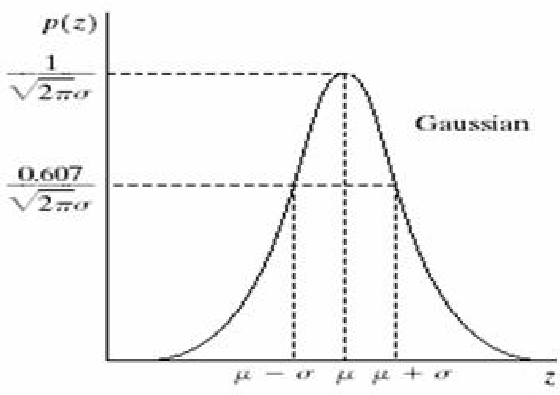


Figure 10.2 Gaussian noise PDF

where z is the gray value, μ is the mean and s is the standard deviation.

Rayleigh Noise

The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} (2/b)/(z-a)e^{-(z-a)^2/b} & for\ z \geq a \\ 0 & for\ z < a \end{cases}$$

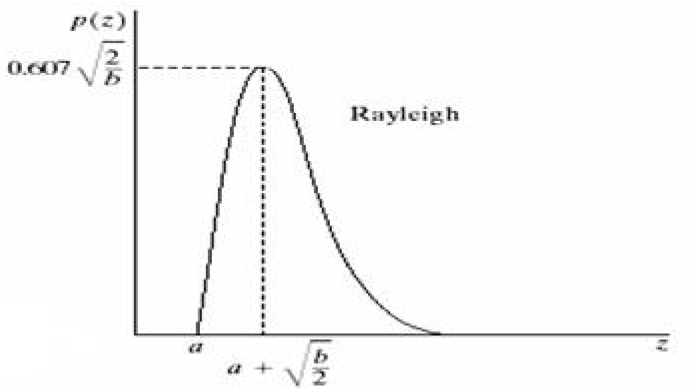


Figure 10.3 Rayleigh noise PDF

Impulse (Salt & Pepper) Noise

The PDF of impulse noise is given by

$$p(z) = \begin{cases} P_a & if\ z = a \\ P_b & if\ z = b \\ 0 & otherwise \end{cases}$$

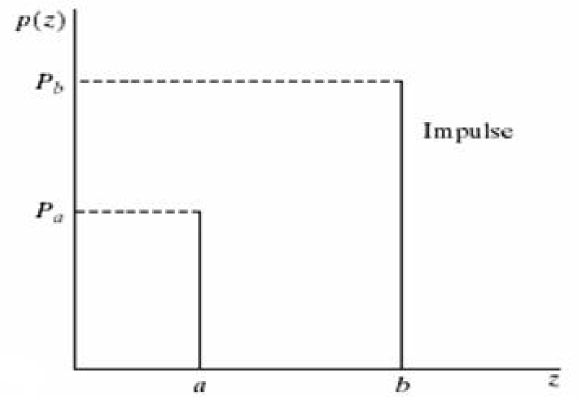


Figure 10.4 Impulse noise PDF

If $b > a$, then gray level b appears as a light dot (salt), otherwise the gray level a appears as a dark dot (pepper).

Determining noise models

The simple image below is well-suited test pattern for illustrating the effect of adding various noise models.

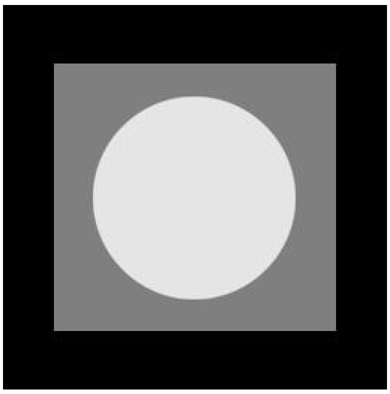


Figure 10.5 Test pattern used to illustrate the characteristics of the noise models

The next figure shows degraded (noisy) images resulted from adding the previous noise models to the above test pattern image.

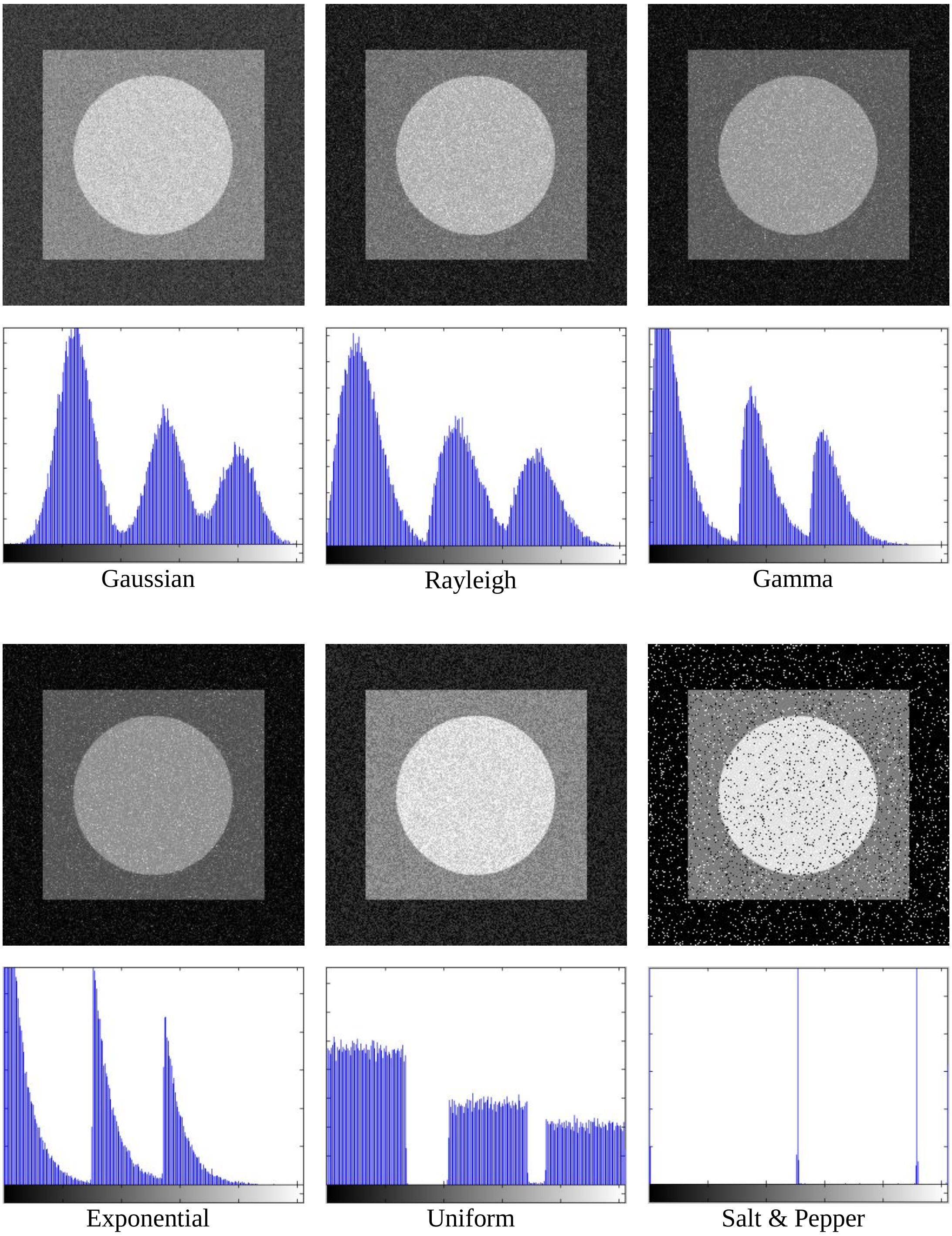


Figure 10.6 Images and histograms from adding Gaussian, Rayleigh, Gamma, Exponential, Uniform, and Salt & Pepper noise.

To determine the noise model in a noisy image, one may select a relatively small rectangular sub-image of relatively smooth region. The histogram of the sub-image approximates the probability distribution of the corrupting model of noise. This is illustrated in the figure below.

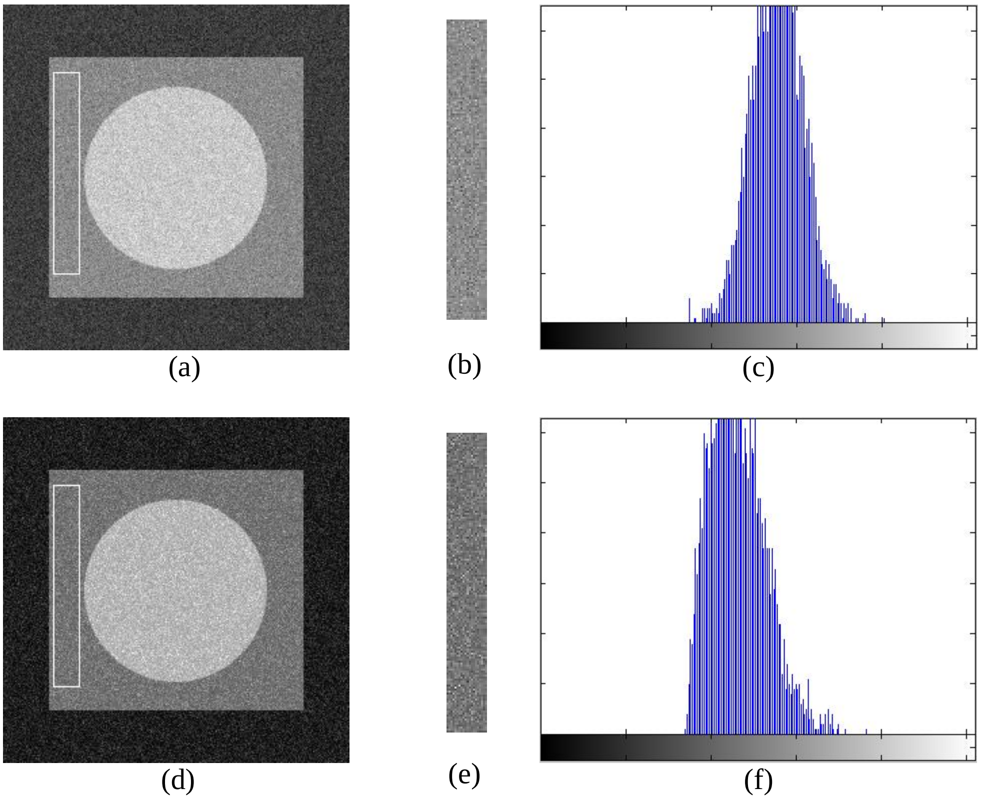


Figure 10.7 (a) Gaussian noisy image. (b) sub-image extracted from a. (c) histogram of b (d) Rayleigh noisy image. (e) sub-image extracted from d. (f) histogram of e

Image restoration in the presence of Noise Only

When the only degradation present in an image is noise, the degradation is modeled as:

$$g(x, y) = f(x, y) + \eta(x, y)$$

and

$$G(u, v) = F(u, v) + N(u, v)$$

Spatial filtering is the method of choice in situations when only additive noise is present. Spatial filters that designed to remove noise include:

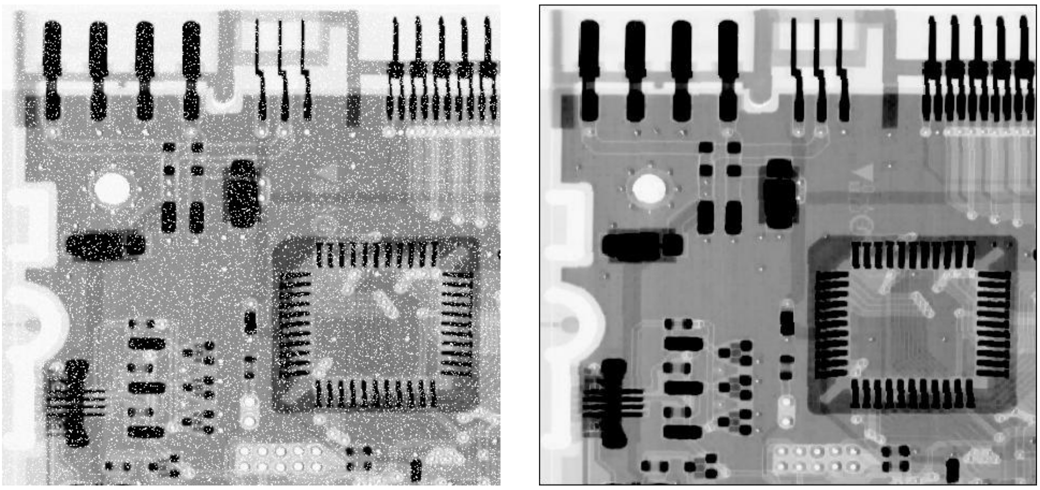
1. *Order Statistics Filters:* e.g. Min, Max, & Median
2. *Adaptive Filters:* e.g. adaptive median filter

Order-Statistics Filters

We have used one of these filters (i.e. median) in the image enhancement. We now use additional filters (min and max) in image restoration.

Min filter

This filter is useful for finding the darkest points in an image. Also, it reduces salt noise as a result of the min operation.

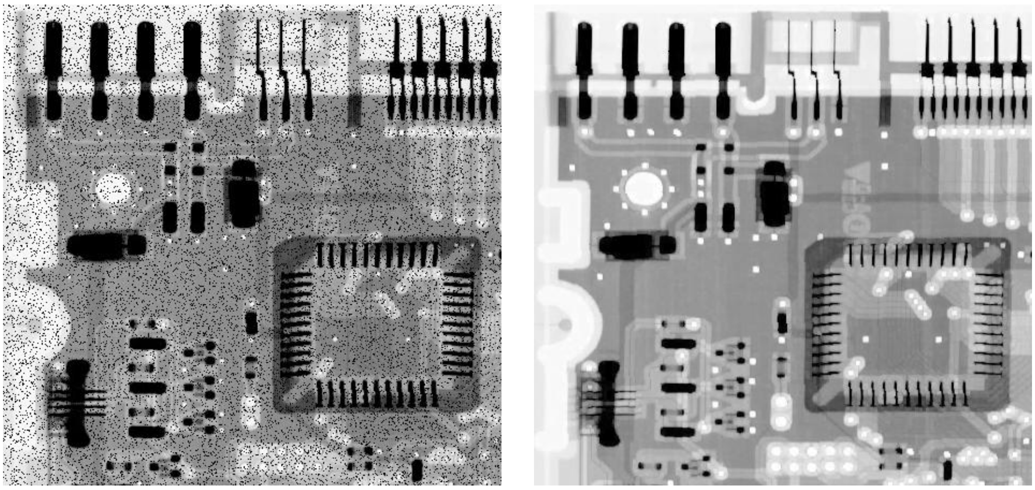


(a) (b)

Figure 10.8 (a) image corrupted by salt noise. (b) Result of filtering (a) with a 3×3 min filter.

Max filter

This filter is useful for finding the brightest points in an image. Also, because pepper noise has very low values, it is reduced by this filter as a result of the max operation.



(a) (b)

Figure 10.9 (a) image corrupted by pepper noise. (b) Result of filtering (a) with a 3×3 max filter.

Adaptive Filters

The previous spatial filters are applied regardless of local image variation. Adaptive filters change their behavior using local statistical parameters in the mask region. Consequently, adaptive filters outperform the non-adaptive ones.

Adaptive median filter

The median filter performs well as long as the spatial density of the impulse noise is not large (i.e. P_a and P_b less than 0.2). Adaptive median filtering can handle impulse noise with probabilities even larger than these. Moreover the adaptive median filter seeks to preserve detail while smoothing non-impulse noise, while the median filter does not do.

The adaptive median filter aims to replace f(x,y) with the median of a neighborhood up to a specified size as long as the median is different from the max and min values but f(x,y)=min or f(x,y)=max. Otherwise, f(x,y) is not changed.

Consider the following notation:

S_{xy} = mask region (neighborhood sub-image)

z_{min} = minimum gray level value in S_{xy}

z_{max} = maximum gray level value in S_{xy}

z_{med} = median of gray levels in S_{xy}

z_{xy} = gray level at coordinates (x, y)

S_{max} = maximum allowed size of S_{xy}

The adaptive median filtering algorithm works in two levels A and B as follows:

Level A: $A1 = z_{med} - z_{min}$

$A2 = z_{med} - z_{max}$

If A1 > 0 AND A2 < 0, Go to level B

Else increase the window size

If window size <= S_{max} repeat level A

Else output z_{med}.

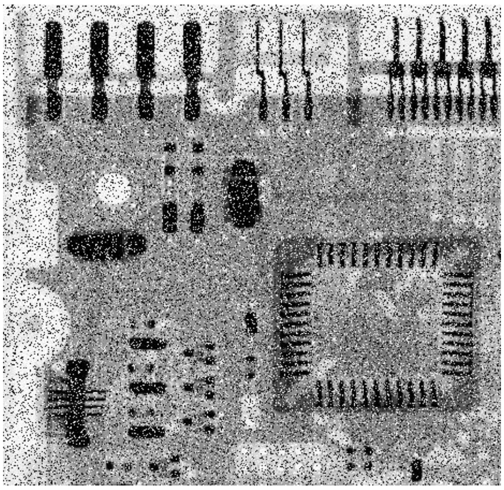
Level B: $B1 = z_{xy} - z_{min}$

$B2 = z_{xy} - z_{max}$

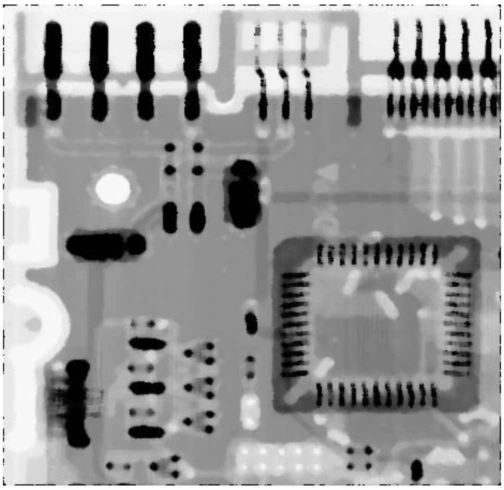
If B1 > 0 AND B2 < 0, output z_{xy}

Else output z_{med}.

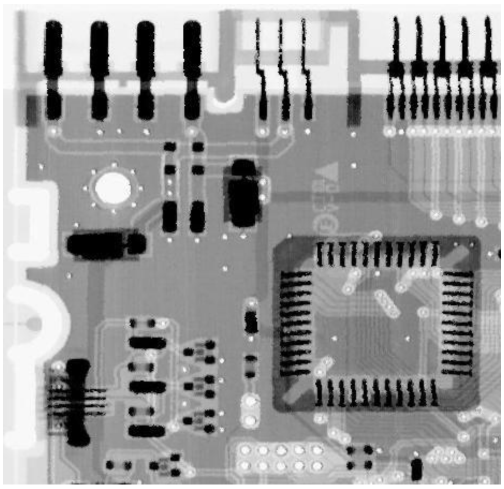
The next figure shows an example of filtering an image corrupted by salt-and-pepper noise with density 0.25 using 7×7 median filter and the adaptive median filter with $S_{max} = 7$.



(a)



(b)



(c)

Figure 10.10 (a) Image corrupted by salt&pepper noise with density 0.25. (b) Result obtained using a 7×7 median filter. (c) Result obtained using adaptive median filter with S_{max} = 7.

From this example, we find that the adaptive median filter has three main purposes:

1. to remove salt-and-pepper (impulse) noise.
2. to provide smoothing of other noise that may not be impulsive.
3. to reduce distortion, such as excessive thinning or thickening of object boundaries.