

(5)

Definition :- Suppose that  $S$  is an ordered set, and  $E \subseteq S$ . If there exists  $b \in S$  such that  $x \leq b$  for every  $x \in E$ , we say that  $E$  is bounded above, and call  $b$  is an upper bound of  $E$ .

ii  $x \geq b$  for every  $x \in E$ , we say that  $E$  is Lower bounded and call  $b$  is an Lower bound of  $E$ .

i.e. (i)  $S$  is order set and  $E \subseteq S$  then  $E$  is bounded above if  $\exists b \in S$  s.t.  $x \leq b \forall x \in E$

(ii)  $S$  is order set and  $E \subseteq S$ , then  $E$  is bounded below if  $\exists b \in S$  s.t.  $b \leq x \forall x \in E$ .

Ex :- (1) Let  $S = \mathbb{Q}$ ,  $E = \{2, 3, 4, 5, 6, 7\}$ .

Then the upper bounds are  $7, 8, 9, 10, \dots$   
and the lower bounds are  $2, -1, 0, -1, -2, \dots$   
 $\therefore E$  is bounded above and bounded below.

(2) Let  $S = \mathbb{Z}$ ,  $E = \{\dots, -3, -2, -1, 0, 1, \}$

Then the upper bound are  $1, 2, 3, 4, \dots$   
since  $1 \in \mathbb{Z}$  and  $1 \geq x \forall x \in E$   
and  $E$  is bounded above.

but  $E$  is not bounded below since  $\nexists b \in \mathbb{Z}$  s.t.