

Definition = An order set  $S$  is said to be least upper bound property if  $E \subseteq S$ ,  $E \neq \emptyset$  and  $E$  bounded above,  $\sup$  (L.u.b) ( $E$ ) exist in  $S$ .

Ex ① Let  $S = \mathbb{R}$ ,  $E = \{x : 0 < x < 1\}$ .

$S = \mathbb{R}$  is least upper bound property

Since

1.  $E \subseteq \mathbb{R}$ .
2.  $E \neq \emptyset$ .
3.  $E$  is bounded above.
4.  $\sup(E) = \text{L.u.b}(E) = 1$  exist in  $\mathbb{R}$ .

②  $S = \mathbb{Q}$ ,  $E = \{x \in \mathbb{Q}, x^2 < 2\}$ .

$E$  does not L.u.b in  $\mathbb{Q}$ .

$\therefore \mathbb{Q}$  does not have least-upper bound property.

Definition = An order set  $S$  is said to be greatest lower bound property, if  $E \subseteq S$ ,  $E \neq \emptyset$ , and  $E$  is bounded below,  $\inf$  (g.l.b.) ( $E$ ) exist in  $S$ .

Ex  $S = \mathbb{R}$ ,  $E = \{x : 0 \leq x \leq 1\}$ .

$\mathbb{R}$  is greatest-lower bound property

Since 1.  $E \subseteq \mathbb{R}$  2.  $E \neq \emptyset$ , 3.  $E$  is bounded below.

4.  $\inf(E) = \text{g.l.b}(E) = 0$  exist in  $\mathbb{R}$ .