# FiLD

**Definition :-** A Field is a Set $F$ with two operation called addition $(+)$ and multiplication $(\cdot)$ which satisfy the Following.

a. $(F, +)$ is Commutitive group.

i.e 1. if $x, y \in F$ then $x+y \in F$.

2. if $x, y \in F$ then $x+y = y+x$.

3. if $x, y, z \in F$ then $(x+y)+z = x+(y+z)$.

4. $\exists$ an element $0 \in F$ s.t $x+0 = 0+x = x \quad \forall x \in F$.

5. For every element $x \in F$, $\exists$ an element $-x \in F$ s.t $x+(-x) = -x+x = 0$

(1-5) is called axiom For addition.

b. $(F - \{0\}, \cdot)$ is Commutitive group.

i.e 1. if $x, y \in F$ then $x \cdot y \in F$.

2. if $x, y \in F$ then $x \cdot y = y \cdot x$

3. if $x, y, z \in F$ then $(x \cdot y) \cdot z = x \cdot (y \cdot z)$.

4. $\exists$ an element $1 \in F$ s.t $x \cdot 1 = 1 \cdot x = x \quad \forall x \in F$

5. For every element $x \in F$, then $\exists$ $x^{-1} \in F$ s.t $x \cdot x^{-1} = x^{-1} \cdot x = 1$

(1-5) called axioin For Multiplication.

c. The distributive Law : if $x, y, z \in F$ then

$x \cdot (y+z) = x \cdot y + x \cdot z$ , $(x+y)z = xz + yz$

Examples:

1. $(\mathbb{R}, +, \cdot)$, $(\mathbb{Q}, +, \cdot)$ are Fields.

2. $(\mathbb{C}, +, \cdot)$ is Field.

3. $(\mathbb{Z}_2, +_2, \cdot_2)$ is Field.

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

| $\cdot$ | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

4. $(\mathbb{Z}, +, \cdot)$ is not Field. (check)

5. $(\mathbb{N}, +, \cdot)$ is not Field. (check).

Definition: $(F, +, \cdot, <)$ is called order Field if

1. $(F, +, \cdot)$ is a Field.
2. $(F, <)$ is an order Set.
3. IF $a \leq b$ then $a + c \leq b + c \quad \forall a, b, c \in F$
4. IF $a, b \in F$, $a > 0$, $b > 0$ then $a \cdot b > 0$.