The density of irrational number

If $a, b \in \mathbb{R}$ s.t $a < b$ then there is K irrational number s.t $a < K < b$.

proof :- Suppose that the theorem is not true.

$\therefore\ \forall K \in \mathbb{R}$ and $a < K < b$, K is rational number.

$\therefore\ a + \sqrt{2} < K + \sqrt{2} < b + \sqrt{2}$ $\quad \sqrt{2}$ غير نسبي

$\therefore\ K + \sqrt{2}$ is irrational number

Then there is no rational number between $a + \sqrt{2}$ and $b + \sqrt{2}$

which is contraduction with density of rational numbers.

$\therefore\ a < K < b$ and K is irrational number.

Corollary :- if $a, b \in \mathbb{R}$ and $a < b$, the set of irrational numbers between a and b is infinite.

proof :- (Exc).