Theorem :-

The Trip $(\mathbb{C},+,\cdot)$ is a Field.

proof :- Let $x=(a,b)$, $y=(c,d)$.

if $x,y\in\mathbb{C}$ then

1. $x+y=(a,b)+(c,d)=(a+c,b+d)\in\mathbb{C}$.

$\therefore\ x+y\in\mathbb{C}$.

2. $x+y=(a,b)+(c,d)=(a+c,b+d)=(c+a,d+b)$
$= y+x$.

$\therefore\ x+y=y+x$.

3- if $x,y,z\in\mathbb{C}$ then $(x+y)+z=((a,b)+(c,d))+(e,f)$
$=(a+c,b+d)+(e,f)$
$=(a+c+e,b+d+f)$
$=(a,b)+(c+e,d+f)=x+(y+z)$

$\therefore\ (x+y)+z=x+(y+z)$.

4. $\exists$ an element $0\in\mathbb{C}$ s.t $x+0=0+x=x$.

We take $0=(0,0)$

$\Rightarrow x+0=(a,b)+(0,0)=(a+0,b+0)=(a,b)=x$

5. For every element $x\in\mathbb{C}$, $\exists -x\in\mathbb{C}$ s.t $x+(-x)=0$.

We take $-x=(-a,-b)$.

$\therefore\ x+(-x)=(a,b)+(-a,-b)=(a+(-a),b+(-b))=(0,0)$