Definition :-

A set A is said to be countable if there exist 1-1 and onto function f from A onto J. ($A \sim J$).

proposition : Every Finite Set is Countable.

Example :-

The set of all integers is Countable.

proof :- Let $f: J \longrightarrow Z$ be a function defined by

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ \frac{-x+1}{2} & \text{if } x \text{ is odd} \end{cases}$$

1- To show that f is 1-1.

Let $a, b \in J$ s-t $a \neq b$.

ⓐ if a, b are even

$\therefore f(a) = \frac{a}{2}$, $f(b) = \frac{b}{2}$ since $a \neq b \Rightarrow \frac{a}{2} \neq \frac{b}{2}$

$\therefore f(a) \neq f(b)$.

ⓑ if a, b are odd.

$f(a) = \frac{-a+1}{2}$, $f(b) = \frac{-b+1}{2}$ since $a \neq b \Rightarrow \frac{-a+1}{2} \neq \frac{-b+1}{2}$

$\therefore f(a) \neq f(b)$.

ⓒ if a is even, b is odd (Exc) ⓓ if a is odd, b is even (Exc)

Thus f is 1-1

2. To show f is onto

$\forall\, b \in Z$ T-p. $\exists\, a \in J$ s-t $f(a) = b$.

(a) if a is even $\Rightarrow f(a) = \frac{a}{2} \Rightarrow \frac{a}{2} = b \Rightarrow a = 2b$.

$\therefore f(a) = \frac{a}{2} = \frac{2b}{2} = b$

$\therefore f(a) = b$.

(b) if a is odd. then $f(a) = \frac{-a+1}{2} \Rightarrow \frac{-a+1}{2} = b$

$\Rightarrow a = -2b+1$

$\therefore f(a) = \frac{-a+1}{2} = \frac{-(-2b+1)+1}{2} = b$.

$\therefore f(a) = 2$.

$\therefore f$ is onto.

$\therefore Z \sim J$.

$\therefore Z$ is Countable.

__Definition__ :- A set A is called at most countable if A either finite or countable.