Chapter two

Metric space الفضاء المتري

<u>Definition</u> :- Let X be any set . A function $d: X \times X \to R$ is called Metric or distance function if it satisfy The following properties :-

(1) $d(x,y) > 0$ iff $x \neq y$

(2) $d(x,y) = 0$ iff $x = y$

(3) $d(x,y) = d(y,x)$ for any $x,y \in X$

(4) $d(x,y) \leq d(x,z) + d(z,y)$ for any $x,y,z \in X$

and the pair (X,d) is called metric space.

<u>Remark</u> :- $d(x,y)$ is called distance.

<u>Examples</u> :- Let $X = R$, $d: X \times X \to R$ be a function defined by $d(x,y) = |x-y|$, then (X,d) is a metric space (is called usuall metric space).

<u>Solution</u> :-

① suppose $d(x,y) > 0$

$\Rightarrow |x-y| > 0 \Rightarrow x \neq y$

$\Leftarrow$ Conversely , suppose $x \neq y$

$\Rightarrow x-y \neq 0$

Thus $|x-y| > 0 \Rightarrow d(x,y) > 0$

② suppose $d(x,y) = 0$

$\Rightarrow |x-y| = 0 \Rightarrow x-y = 0 \Rightarrow x = y$

$\Leftarrow$ Conversely , suppose $x = y$

$\Rightarrow x-y = 0 \Rightarrow x-y = 0 \Rightarrow |x-y| = 0$

$\Rightarrow d(x,y) = 0$