

Definition :- Let (X, d) be a metric space and let r be a positive number a neighbourhood of a point $p \in X$, the set of all points $x \in X$ $\exists d(p, x) < r$ and is denoted by $N_r(p)$

$$\therefore N_r(p) = \{x \in X : d(p, x) < r\}$$

Example :- let (\mathbb{R}, d) be the usual metric space $r=5$,
 $p=2$

sol:

$$N_r(p) = \{x \in X : d(p, x) < r\}$$

$$N_5(2) = \{x \in \mathbb{R} : d(2, x) < 5\}$$

$$= \{x \in \mathbb{R} : |x - 2| < 5\}$$

$$= \{x \in \mathbb{R} : -5 < x - 2 < 5\}$$

$$= \{x \in \mathbb{R} : -3 < x < 7\}$$



Exc Find $N_3(-2)$

Example 1:- Let (\mathbb{R}^2, d) be usual metric space.
Describe the neighbourhood of $(0,0)$ with radius 1.

Solution:-

$$\begin{aligned} N_1(0,0) &= \{(x,y) \in \mathbb{R}^2 : d(0,0)(x,y) < 1\} \\ &= \{(x,y) \in \mathbb{R}^2 : \sqrt{(y-0)^2 + (x-0)^2} < 1\} \\ &= \{(x,y) \in \mathbb{R}^2 : \sqrt{y^2 + x^2} < 1\} \\ &= \{(x,y) \in \mathbb{R}^2 : y^2 + x^2 < 1\} \end{aligned}$$

Example 2:- Let (\mathbb{R}^2, d) be usual metric space where d is a metric on \mathbb{R}^2 defined by $d(x_1, x_2)(y_1, y_2) = |y_1 - x_1| + |y_2 - x_2|$. Describe the neighbourhood of $(0,0)$ with radius 1.

Solution:-

$$\begin{aligned} N_1(0,0) &= \{(x,y) \in \mathbb{R}^2 : d(0,0)(x,y) < 1\} \\ &= \{(x,y) \in \mathbb{R}^2 : |x-0| + |y-0| < 1\} \\ &= \{(x,y) \in \mathbb{R}^2 : |x| + |y| < 1\} \end{aligned}$$