

⑨

Corollary:- Let  $(X, d)$  be a metric space. Then  $A$  is open iff  $A = \bigcup_{x \in A} N_r(x)$ .

Proof:  $\Rightarrow$  Suppose  $A = \bigcup_{x \in A} N_r(x)$

T.P  $A$  is open

Since  $N_r(x)$  is an open set and Union of Family open set is open, Then  $A$  is open.

$\Leftarrow$  Conversely  $A$  is open

T.P  $A = \bigcup_{x \in A} N_r(x)$

Since  $A$  is open, Then  $\forall x \in A \exists r > 0 \ni N_r(x) \subseteq A$

$$\Rightarrow \bigcup_{x \in A} N_r(x) \subseteq A \quad \text{--- ①}$$

$$\{x\} \subseteq N_r(x) \Rightarrow \bigcup_{x \in A} \{x\} \subseteq \bigcup_{x \in A} N_r(x)$$

$$A \subseteq \bigcup_{x \in A} N_r(x) \quad \text{--- ②}$$

From ① and ② we get  $A = \bigcup_{x \in A} N_r(x)$

Definition:- Let  $(X, d)$  be a metric space,  $E \subseteq X$  a point  $p \in X$  is called limit point of  $E$  if every neighbourhood of  $p$  ( $N_r(p) - \{p\}$ )  $\cap E \neq \emptyset$ .

Example:- Let  $(\mathbb{R}, d)$  be a usual metric space  
 $A = (2, 3]$

2 is limit point of  $A$  because every  $N_r(2) - \{2\} \cap A \neq \emptyset$

0 is not limit point of  $A$  because  $\exists N_r(0) - \{0\} \cap A = \emptyset$