(Numerical sequnee)

convergent sequnee.

Definition:- let (X, d) be a metric space and $a \in X$ a sequnce $\langle a_n \rangle$ is said to be converge to a if for each $\epsilon > 0$, there exists a positive integer N $\ni d(a_n, a) \leq \epsilon \quad \forall\, n \geq N$.

if $\langle a_n \rangle$ does not converges then it is called diverges.

Remarks:-

1- $\langle a_n \rangle$ converges to a also means a is Limit point of $\langle a_n \rangle$ and we written $a_n \to a$ or $\lim\limits_{n \to \infty} a_n = a$.

2- The set $\{a_1, a_2, a_3, \dots\}$ is called the range of $\langle a_n \rangle$

Definition:- let (X, d) be a metric space and $a \in X$. A sequnce $\langle a_n \rangle$ is called bounded if its range is bounded set.

Example(1):-

Let (R, d) be usual metric space, show that $\langle \frac{1}{n} \rangle$ converges to 0.

proof:- 1. let $\epsilon > 0$ be given. $\quad \forall\, n \geq N$.

2. T.P. $\exists$ positive integer $N \ni d(a_n, a) < \epsilon$

i.e $\exists$ positive integer $N \ni |a_n - a| < \epsilon \quad \forall\, n \geq N$.

3. let N be smallest positive integer $\ni N > \frac{1}{\epsilon}$

since $n \geq N > \frac{1}{\epsilon} \Rightarrow \frac{1}{n} \leq \frac{1}{N} < \epsilon$

$$\Rightarrow \left|\frac{1}{n}\right| < \epsilon \Rightarrow \left|\frac{1}{n} - 0\right| < \epsilon$$

$$\Rightarrow d(a_n, a) < \epsilon$$

$\therefore \langle a_n \rangle$ is converges to 0

The sequnce $\langle \frac{1}{n} \rangle$ is bounded

since the range of $\langle \frac{1}{n} \rangle$ is $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$.

it is bounded $\Rightarrow$ The sequnce $\langle \frac{1}{n} \rangle$ is bounded sequnce.

Example ②:- show that the sequnce $\langle \frac{1}{n+1} \rangle$ converges to 0.

proof :- 1. let $\epsilon > 0$ be given

2. T.p $\exists$ positive integer $N \ni \left|\frac{1}{n+1} - 0\right| < \epsilon \ \forall n \geq N$.

3. let N be smallest positive integer $\ni N > \frac{1}{\epsilon} - 1$.

since $n \geq N > \frac{1}{\epsilon} - 1 \Rightarrow n > \frac{1}{\epsilon} - 1 \Rightarrow n+1 > \frac{1}{\epsilon}$

$$\Rightarrow \frac{1}{n+1} < \epsilon \Rightarrow \left|\frac{1}{n+1} - 0\right| < \epsilon \Rightarrow d(a_n, a) < \epsilon \quad \forall n \geq N$$

$\therefore \langle \frac{1}{n+1} \rangle$ is converges to 0.

The sequnce $\langle \frac{1}{n+1} \rangle$ is bounded, since the range of $\langle \frac{1}{n+1} \rangle$ is $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$ which is bounded set