Theorem :- Let $\langle a_n \rangle$ be a sequnee in a metric space (X, d), if $a_1, a_2 \in X$, and $\langle a_n \rangle$ converges to a_1, a_2 Then $a_1 = a_2$.

Proof :- since $\langle a_n \rangle$ converges to a_1

$\Rightarrow \exists$ positive integer $N_1 \ni d(a_n, a_1) < \frac{\epsilon}{2} \ \forall n \geq N_1$

since $\langle a_n \rangle$ converges to a_2

$\Rightarrow \exists$ positive integer $N_2 \ni d(a_n, a_2) < \frac{\epsilon}{2} \ \forall n \geq N_2$

let $N = \max\{N_1, N_2\}$.

$\therefore d(a_n, a_1) < \frac{\epsilon}{2}, \quad d(a_n, a_2) < \frac{\epsilon}{2} \quad \forall n \geq N$.

$$d(a_1, a_2) \leq d(a_1, a_n) + d(a_n, a_2)$$
$$< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

$\therefore d(a_1, a_2) < \epsilon$

$\Rightarrow d(a_1, a_2) = 0 \Rightarrow a_1 = a_2$.

Definition :- let (X, d) be a metric space and let y be a fixed point in X. ; a subset E of X is called bounded if $\exists$ a positive real number $M \ni d(x, y) \leq M$ for all point $x \in X$.