Theorem :-

suppose $\langle a_n \rangle$, $\langle b_n \rangle$ are two real sequnce and $\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$, then

(a) $\lim_{n\to\infty} (a_n + b_n) = a + b$

(b) $\lim_{n\to\infty} (K \cdot a_n) = K a$, $\lim_{n\to\infty} (k + a_n) = K + a$

(c) $\lim_{n\to\infty} \frac{1}{a_n} = \frac{1}{a}$, $(a_n \neq 0, a \neq 0$, for $n = 1, 2, \ldots \}$.

(d) $\lim_{n\to\infty} (a_n \cdot b_n) = a \cdot b$.

proof :- (a) let $\epsilon > 0$ be given.

Since $\langle a_n \rangle \longrightarrow a$ $(\lim_{n\to\infty} a_n = a)$

$\Rightarrow \exists$ positive integer $N_1 \ni |a_n - a| < \frac{\epsilon}{2}$ $\forall n \geq N_1$

Since $\lim_{n\to\infty} b_n = b$

$\Rightarrow \exists$ positive integer $N_2 \ni |b_n - b| < \frac{\epsilon}{2}$ $\forall n \geq N_2$.

let $N = \max\{N_1, N_2\}$.

$$\therefore |(a_n + b_n) - (a + b)| = |(a_n - a) + (b_n - b)|$$
$$\leq |a_n - a| + |b_n - b|$$
$$< \frac{\epsilon}{2} + \frac{\epsilon}{2}$$

$\therefore |(a_n + b_n) - (a + b)| < \epsilon$.

$\therefore \lim_{n\to\infty} (a_n + b_n) = a + b$