Example:- Let $(\mathbb{R}, d)$ be a usual metric space, show that the sequence $\langle \frac{1}{n+1} \rangle$ is cauchy.

Solution:- 1. let $\epsilon > 0$ be given

2. let N be smallest positive integer $\ni N > \frac{2}{\epsilon} - 1$

$\forall n \geq N, m \geq N$.

since $n \geq N > \frac{2}{\epsilon} - 1$

$$\Rightarrow n > \frac{2}{\epsilon} - 1 \Rightarrow n+1 > \frac{2}{\epsilon} \Rightarrow \frac{1}{n+1} < \frac{\epsilon}{2}$$

$$\Rightarrow \left| \frac{1}{n+1} \right| < \frac{\epsilon}{2}.$$

by the same way $\left| \frac{1}{m+1} \right| < \frac{\epsilon}{2}$.

$$\therefore d(a_{n+1}, a_{m+1}) = \left| \frac{1}{n+1} - \frac{1}{m+1} \right| \leq \left| \frac{1}{n+1} \right| + \left| \frac{1}{m+1} \right|$$

$$< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

$$\therefore d(a_{n+1}, a_{m+1}) < \epsilon$$

$\therefore \left| \frac{1}{n+1} - \frac{1}{m+1} \right| < \epsilon \Rightarrow \langle \frac{1}{n+1} \rangle$ is Cauchy sequence.

Theorem:- In any metric space, every convergent sequence is a cauchy sequence.

Proof:- let (X, d) be a metric space.

let $\langle a_n \rangle$ be convergent sequence to a

let $\epsilon > 0$ be given.