since $\langle a_n \rangle$ converges to a

$\Rightarrow \exists$ positive integer $N \ni d(a_n, a) < \frac{\epsilon}{2} \quad \forall n \geq N$.

$\therefore d(a_n, a_m) \leq d(a_n, a) + d(a, a_m)$

$$< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

$\therefore d(a_n, a_m) < \epsilon$

$\therefore \langle a_n \rangle$ is a Cauchy sequnce.

<u>Remark①:-</u> The converse of above theorem is not true in general.

<u>Example:-</u> let $(\mathbb{R} - \{0\}, d)$ be a usual metric space. The sequnce $\langle \frac{1}{n} \rangle$ is cauchy sequnce in a metric space $(\mathbb{R} - \{0\}, d)$, but is not convergent sequnce.

Since $\langle \frac{1}{n} \rangle$ converges to zero and $0 \notin (\mathbb{R} - \{0\}, d)$

Therefore $\langle \frac{1}{n} \rangle$ is diverges.

<u>Rermark②</u> A bounded sequnce is not necessary to be convergent sequnce.

<u>Example:</u> (Exe).