Theorem (1-5):- Every finite integral domain is field .

Proof :-

Let $(R,+,.)$ be a finite integral domain .

< **T-P $(R,+,.)$ is a field** >?

$\because (R,+,.)$ be a finite integral domain

$\therefore (R,+,.)$is a commutative ring with identity and has no zero divisors $\cdots\cdots *$ and $(R,+,.)$ has n distinct elements

let $a \in R$ and $a \neq 0$ < **T-p , a is an invertible element** > ?

Consider the set $R^* = \{ a.x , \forall x \in R$ and $x \neq 0 \}$

$\therefore \forall y \in R^*$, $y \neq 0$

If $a.x = a.y$

$\Rightarrow a.x - a.y$

$\Rightarrow a.(x - y) = 0$

since , R has no zero divisors and $a \neq 0$

$\Rightarrow (x - y) = 0$

$\Rightarrow x = y$

$\therefore$, R^* containing all distinct non zero elements of R $\overset{(i.e)}{\Longrightarrow} R^* = R - \{0\}$

Since, R is a ring with identity حسب المعطى $\Rightarrow 1 \in R$

$\Rightarrow 1 \in R^*$ (since $R^* = R - \{0\}$)

$\Rightarrow 1 = a.x$, for some $x \in R$

$\Rightarrow a^{-1} = x$.

Then , each element of R has multiplicative inverse $\cdots\cdots **$

Therefore , by $*$ and $**$ we get $(R,+,.)$ is field . □