Prove that if $x \in w_p$ then l is unique. Show also that w_p is a linear space with the usual operations $\lambda(x_k) = (\lambda x_k)$, $(x_k) + (y_k) = (x_k + y_k)$.

Define

$$\|x\| = \sup_n \left(n^{-1} \sum_{k=1}^{n} |x_k|^p \right)^{1/p} \quad (1 \leqslant p < \infty),$$

$$\|x\| = \sup_n n^{-1} \sum_{k=1}^{n} |x_k|^p \quad (0 < p < 1).$$

Show that $\|x\|$ is a norm when $p \geqslant 1$ and a p-norm when $0 < p < 1$. Prove also that w_p is a Banach space when $p \geqslant 1$ and a complete p-normed space when $0 < p < 1$.

We shall be examining some further properties of the space w_p in chapter 7.

13. Let $X \neq \{\theta\}$ be a normed space. Prove that X is a Banach space if and only if $\{x \in X \mid \|x\| = 1\}$ is complete.