CH- 5
THICK LENSES

5.6 GENERAL THICK-LENS FORMULAS

A set of formulas that can be used for the calculation of important constants generally associated with a thick lens is presented below in the form of two equivalent sets.

Gaussian formulas	*Power formulas*	
$\frac{n}{f} = \frac{n'}{f_1'} + \frac{n''}{f_2''} - \frac{dn''}{f_1' f_2''} = \frac{n''}{f''}$	$P = P_1 + P_2 - \frac{d}{n'} P_1 P_2$	(5g)
$A_1F = -f\left(1 - \frac{d}{f_2'}\right)$	$A_1F = -\frac{n}{P}\left(1 - \frac{d}{n'} P_2\right)$	(5h)
$A_1H = +f\frac{d}{f_2'}$	$A_1H = +\frac{n}{P}\frac{d}{n'} P_2$	(5i)
$A_2F'' = +f''\left(1 - \frac{d}{f_1'}\right)$	$A_2F'' = +\frac{n''}{P}\left(1 - \frac{d}{n'} P_1\right)$	(5j)
$A_2H'' = -f''\frac{d}{f_1'}$	$A_2H'' = -\frac{n''}{P}\frac{d}{n'} P_1$	(5k)

These equations are derived from geometrical relations that can be obtained from a diagram like Fig. 5G. As an illustration, the gaussian equation (5k) is derived as follows. From the two similar right triangles $T_1A_1F'_1$ and $T_2A_2F'_1$, we can write corresponding sides as proportions

$$\frac{A_1F_1'}{A_1T_1} = \frac{A_2F_1'}{A_2T_2} \quad \text{or} \quad \frac{f_1'}{h} = \frac{f_1' - d}{j}$$

and, from the two similar right triangles $N''H''F''$ and T_2A_2F'', we can write the proportions

$$\frac{H''F''}{H''N''} = \frac{A_2F''}{A_2T_2} \quad \text{or} \quad \frac{f''}{h} = \frac{f'' - H''A_2}{j}$$

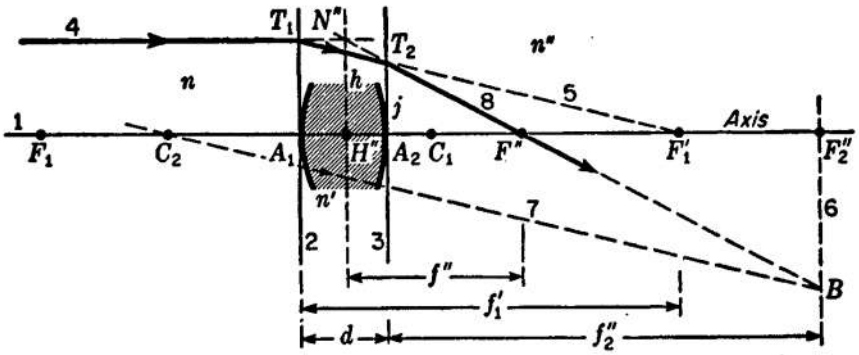

FIGURE 5G: The oblique-ray method for graphically tracing paraxial rays through a thick lens.

If we solve each equation for *j/h* and then equate the right-hand sides of the resultant equations, we obtain

$$\frac{f_1' - d}{f_1'} = \frac{f'' - H''A_2}{f''} \qquad \text{or} \qquad H''A_2 = f''\frac{d}{f_1'}$$

If we now reverse the segment $H''A_2$ to A_2H'' by changing the sign from + to -, we obtain

$$A_2H'' = -f''\frac{d}{f_1'}$$

In terms of surface power and lens power,

$$P_1 = \frac{n}{f_1} = \frac{n'}{f_1'} \qquad P_2 = \frac{n'}{f_2'} = \frac{n''}{f_2''} \qquad P = \frac{n}{f} = \frac{n''}{f''} \qquad (5l)$$

the same equation can be written

$$A_2H'' = -\frac{n''}{P}\frac{d}{n'}P_1$$

In the design of certain optical systems it is convenient to know the *vertex power* of a lens. This power, sometimes called *effective power,* is given as

$$P_v = \frac{P}{1 - dP_1/n'} \qquad (5m)$$

and is defined as the reciprocal of the distance from the back surface of the lens to the secondary focal point. This distance is commonly called the *back local length.*
Since $P_v = 1/A_2F''$, the above equation for vertex power is obtained by inverting Eq. (5j). In the inversion the lens is assumed to be in air so that $n'' = 1$.

In a similar way the distance from the primary focal point to the front surface of the lens is called the *front focal length,* and the reciprocal of this distance is called the *neutralizing power. Pn = 1/A₁F.* Calling P_n the neutralizing power, we can take the reciprocal of Eq. (5h) to obtain

$$P_n = \frac{P}{1 - dP_2/n'} \qquad (5n)$$

The name is derived from the fact that a thin lens of this specified power and of opposite sign will, upon contact with the front surface, give zero power to the combination.

The following example will serve as an illustration of the use of thick-lens formulas applied to two surfaces.

EXAMPLE 2: A lens has the following specifications: $r_1 = +1.5$ cm, $r_2 = +1.5$ cm, $d = 2.0$ cm, $n = 1.00$, $n' = 1.60$, and $n'' = 1.30$. Find *(a)* the primary and secondary focal lengths of the separate surfaces, *(b)* the primary and secondary focal lengths of the system, and *(c)* the primary and secondary principal points.

SOLUTION: *(a)* To apply the gaussian formulas, we first calculate the individual focal lengths of the surfaces by means of Eq. (5f).

$$\frac{n}{f_1} = \frac{n' - n}{r_1} = \frac{1.60 - 1.00}{1.5} \qquad f_1 = \frac{1.00}{0.40} = +2.50 \text{ cm}$$

$$= 0.400 \qquad f_1' = \frac{1.60}{0.40} = +4.00 \text{ cm}$$

$$\frac{n'}{f_2'} = \frac{n'' - n'}{r_2} = \frac{1.30 - 1.60}{1.5} \qquad f_2' = \frac{1.60}{-0.20} = -8.00 \text{ cm}$$

$$= -0.200 \qquad f_2'' = \frac{1.30}{-0.20} = -6.50 \text{ cm}$$

(b) The focal lengths of the system are calculated from Eq. (5g).

$$\frac{n}{f} = \frac{n'}{f_1'} + \frac{n''}{f_2''} - \frac{d}{f_1'}\frac{n''}{f_2''} = \frac{1.60}{4.00} + \frac{1.30}{-6.50} - \frac{2.00}{4.00}\frac{1.30}{-6.50}$$

$$\frac{n}{f} = 0.40 - 0.20 + 0.10 = 0.30$$

$$\text{or } f = \frac{1.00}{0.30} = +3.333 \text{ cm} \quad \text{and} \quad f'' = \frac{n''}{0.30} = \frac{1.30}{0.30} = +4.333 \text{ cm}$$

The focal points of the system are given by Eqs. (5h) and (5j).

$$A_1F = -f\left(1 - \frac{d}{f_2'}\right) = -3.333\left(1 - \frac{2.0}{-8.0}\right) = -4.166 \text{ cm}$$

$$A_2F'' = +f''\left(1 - \frac{d}{f_1'}\right) = +4.33\left(1 - \frac{2.0}{4.0}\right) = +2.167 \text{ cm}$$

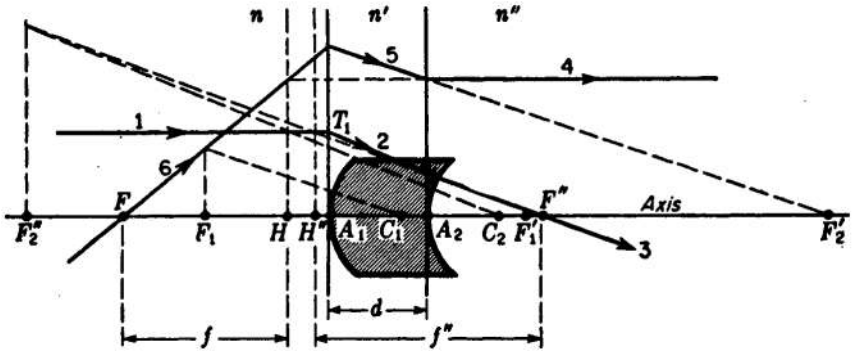

FIGURE 5H: A graphical construction for locating the focal points and principal points of a thick lens.

(c) The principal points are given by Eqs. (5i) and (5k).

$$A_1H = +f\frac{d}{f_2'} = +3.33\frac{2.0}{-8.0} = -0.833 \text{ cm}$$

$$A_2H'' = -f''\frac{d}{f_1'} = -4.33\frac{2.0}{4.0} = -2.167 \text{ cm}$$

Positive signs represent distances measured to the right of the reference vertex and negative signs those measured to the left.

By subtracting the magnitudes of the two intervals A_1F and A_1H, the primary focal length FH = 4.166 - 0.833 = 3.333 cm is obtained and serves as a check on the

calculations in part *(b)*. Similarly the addition of the two intervals A_2F'' and A_2H'' gives the secondary focal length

$$H''F'' = 2.167 + 2.167 = 4.334 \text{ cm}$$

The graphical solution of this same problem is shown in Fig. 5H. After the axis is drawn and the lens vertices A_1 and A_2 and the centers C_1 and C_2 are located, the individual focal points F_1, F'_1, F'_2, and F''_2 are laid off according to the results in part *(a)*. The parallel ray 1 is refracted at the first surface toward F'_1. The oblique-ray method is applied to this ray 2 at the second surface, and the final ray 3 is obtained. The point where ray 3 crosses the axis locates the secondary focal point F'', and the point where its backward extension intersects ray 1 locates the secondary principal plane H''. Ray 4 is constructed backward by drawing it parallel to the axis and from right to left. The first refraction gives ray 5 up and to the left as if it came from F'_2. The oblique-ray method applied to ray 5 at the left-hand surface yields ray 6. The point where ray 6 crosses the axis locates F, and the point where it crosses the extension of ray 4 locates H. Hence parts *(b)* and *(c)* of the problem are solved graphically, and they check with the calculated values.

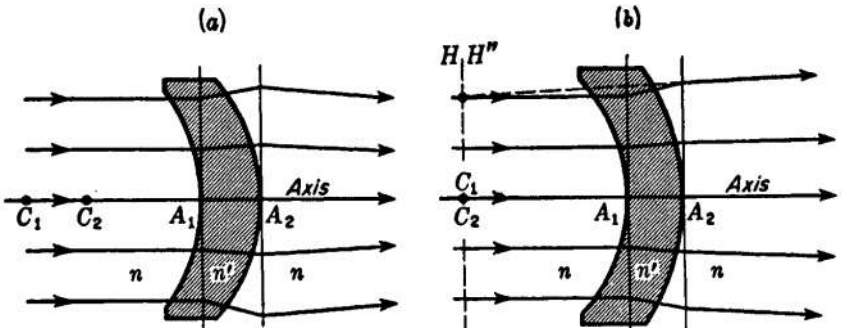

FIGURE 51: Special thick lenses; *(a)* a positive lens with equal radii of curvature; *(b)* a negative lens with concentric surfaces.