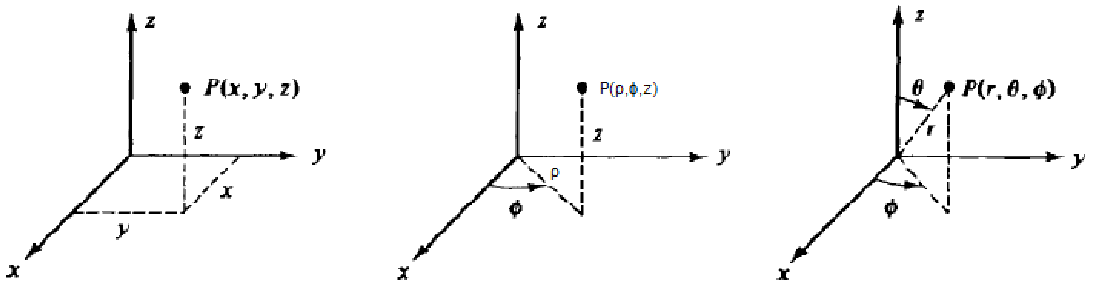
1. 3 Coordinate Systems

Actually different kinds of coordinates systems usually used in physics. The most often used ones are *Cartesian*, *Cylindrical*, and *Spherical*. A point P may described by these three coordinates as shown in the figure below;



(*a*) Cartesian (*b*) Cylindrical (*c*) Spherical

The order of specifying the coordinates is important an should carefully be followed. The angle φ is the same in both cylindrical an spherical systems. But in the order of the coordinates, φ appears in the second position in cylindrical (r, φ, z) and third position in spherical (r, θ, φ). The same symbol r is used in both cylindrical and spherical coordinates but for two quit different things. In cylindrical coordinate r measure the distance from z-axis to a plane normal to the xy-plane. While in the spherical system r measure the distance from the origin to the point p(r, θ, φ).

Remarks:

1) The domain of the coordinates are as follows;

Cartesian	Cylindrical	Spherical
$-\infty \leq x \leq +\infty$	$0 \leq r \leq \infty$	$0 \leq r \leq \infty$
$-\infty \leq y \leq +\infty$	$0 \leq \varphi \leq 2\pi$	$0 \leq \theta \leq \pi$
$-\infty \leq z \leq +\infty$	$-\infty \leq z \leq +\infty$	$0 \leq \varphi \leq 2\pi$

2) The transformation equations from cylindrical to Cartesian coordinates are;

$$x = r\ cos\theta$$

$$y = r\ sin\theta$$

$$z = z$$

3) The transformation equations from spherical to Cartesian coordinates are;

$$x = r\ sin\theta\ cos\varphi$$

$$y = r\ sin\theta\ sin\varphi$$

$$z = r\ cos\theta$$

4) The components forms of a vector in three systems are;

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \qquad (Carteisan)$$

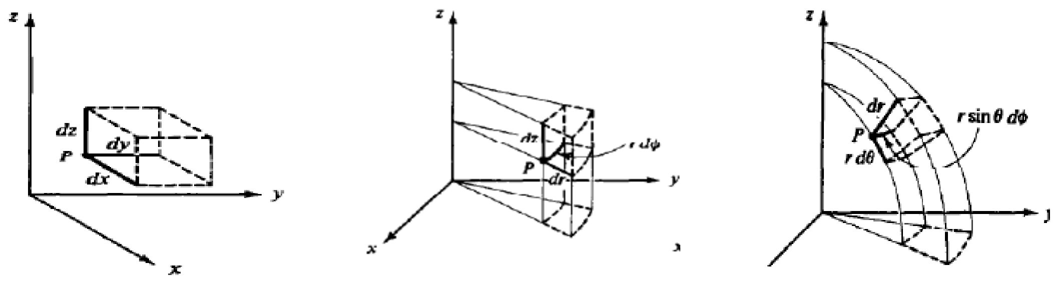
$$\vec{A} = A_r\hat{r} + A_\varphi\hat{\varphi} + A_z\hat{k} \qquad (Cylindrical)$$

$$\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\varphi\hat{\varphi} \qquad (Spherical)$$

Question: What will happened when a point p (x, y, z) is expanded to (x+dx, y+dy, z+dz) or (r+dr, φ+dφ, z+dz) or (r+dr, θ+dθ, φ+dφ)?

Answer:

As show in the figures;



1) A differential volume dV is formed;

$$dV = dxdydz \qquad \text{(Cartesian)}$$

$$dV = rdrd\varphi dz \qquad \text{(Cylindrical)}$$

$$dV = r^2 sin\theta dr d\theta d\varphi \quad \text{(Spherical)}$$

2) A differential area dS is formed;

$$ds = dxdy, = dxdz, = dydz \quad \text{(Cartesian)}$$

$$ds = drdz, = rdrd\varphi, = rd\varphi dz \quad \text{(Cylindrical)}$$

$$ds = rdrd\theta, = r^2 sin\theta d\theta d\varphi, = rsin\theta drd\varphi \quad \text{(Spherical)}$$

3) A differential line dℓ is formed;

$$d\vec{l} = dx\hat{\imath} + dy\hat{\jmath} + dz\vec{k} \quad \text{(Cartesian)}$$

$$d\vec{l} = dr\hat{r} + rd\varphi\hat{\varphi} + dz\hat{z} \quad \text{(Cylindrical)}$$

$$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + rsin\theta d\varphi\hat{\varphi} \quad \text{(Spherical)}$$

However;

$$dl^2 = dx^2 + dy^2 + dz^2 \quad \text{(Cartesian)}$$

$$dl^2 = dr^2 + r^2 d\varphi^2 + dz^2 \quad \text{(Cylindrical)}$$

$$dl^2 = dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2 \quad \text{(Spherical)}$$

H.W.: What are the forms of $\vec{\nabla}$ and $\vec{\nabla}^2$in these three coordinates systems?