Newton Divided Difference Interpolation Polynomial (NDDIP) for equal spacing

* تستخدم هذه الطريقة عندما يكون الفرق بين القيم المتتالية لـ (X) مقدار ثابت (h).

$$X_{i+1} - X_i = h \text{ (constant)}$$

* تسمى هذه الطريقة ايضاً "طريقة Newton-Gregory".

* هنالك اسلوبان لهذه الطريقة اعتماداً على موقع (X) المطلوب اجراء الاستكمال عندها:

1. Newton forward difference:

* عندما تقع قيمة (X) المطلوبة في النصف الاول في جدول البيانات
* تكون متعددة الحدود $[P_n(x)]$ بالصيغة التالية:

$$P_n(x) = y_0 + K\frac{\Delta y_0}{1!} + K(K-1)\frac{\Delta^2 y_0}{2!} + K(K-1)(K-2)\frac{\Delta^3 y_0}{3!} + \cdots$$

where: $K = \dfrac{x - x_0}{h}$, Δ : Forward operator (عامل الفروقات الامامية)

* نعتبر ان لدينا $(n+1)$ نقطة.

* Difference table:

If we have (5) points for example $(n=4)$:

x_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$
x_0	y_0				
		Δy_0: $y_1 - y_0$			
x_1	y_1		$\Delta^2 y_0$: $\Delta y_1 - \Delta y_0$		
		Δy_1: $y_2 - y_1$		$\Delta^3 y_0$: $\Delta^2 y_1 - \Delta^2 y_0$	
x_2	y_2		$\Delta^2 y_1$: $\Delta y_2 - \Delta y_1$		$\Delta^4 y_0$: $\Delta^3 y_1 - \Delta^3 y_0$
		Δy_2: $y_3 - y_2$		$\Delta^3 y_1$: $\Delta^2 y_2 - \Delta^2 y_1$	
x_3	y_3		$\Delta^2 y_2$: $\Delta y_3 - \Delta y_2$		
		Δy_3: $y_4 - y_3$			
x_4	y_4				

Example:- Use NDDIP to find (y) at $x = 8$ from the following data:-

x	0	5	10	15	20	25
y	7	11	14	18	24	32

Solution:

$x = 8$ is in the first half, therefore we use the forward difference:

$$P_5(x) = y_0 + K\Delta y_0 + K(K-1)\frac{\Delta^2 y_0}{2!} + K(K-1)(K-2)\frac{\Delta^3 y_0}{3!} + K(K-1)(K-2)(K-3)\frac{\Delta^4 y_0}{4!} + K(K-1)(K-2)(K-3)(K-4)\frac{\Delta^5 y_0}{5!}$$

$$h = X_{i+1} - X_i = 5, \quad K = \frac{X - X_0}{h} \Rightarrow K = \frac{X}{5} \Rightarrow K = \frac{8}{5} = 1.6 .$$

Difference table:

X_i	y_i	Δy_i	$\Delta^2 y_i$	$\Delta^3 y_i$	$\Delta^4 y_i$	$\Delta^5 y_i$
0	7					
		Δy_0 4				
5	11		$\Delta^2 y_0$ −1			
		3		$\Delta^3 y_0$ 2		
10	14		1		$\Delta^4 y_0$ −1	
		4		1		$\Delta^5 y_0$ 0
15	18		2		−1	
		6		0		
20	24		2			
		8				
25	32					

(3)

$$P_5(8) = 7 + (1.6)(4) - \frac{1}{2}(1.6)(1.6-1) + \frac{2}{6}(1.6)(0.6)(-0.4) - \frac{1}{24}(1.6)(0.6)(-0.4)(-1.4) + \frac{0}{120}(1.6)(0.6)(-0.4)*(-1.4)(2.4)$$

$$P_5(8) = 12.769$$

∴ The interpalated point (x, y) is :

$x = 8$, $y = 12.769$

2. Newton Backward Difference :-

* تستخدم هذه الطريقة عندما تقع قيمة (X) المطلوب استكمالها في النصف الثاني في جدول البيانات .

* تكتب متعددة الحدود كما يلي :

$$P_n(x) = y_n + \frac{\nabla y_n}{1!}K + \frac{\nabla^2 y_n}{2!}K(K+1) + \frac{\nabla^3 y_n}{3!}K(K+1)(K+2) + \cdots$$

where $K = \frac{x - x_n}{h}$, ∇ : Backward Difference operator

*Difference table

x_i	y_i	∇y_i	$\nabla^2 y_i$	$\nabla^3 y_i$	$\nabla^4 y_i$
x_0	y_0				
		$\nabla y_1 = y_1 - y_0$			
x_1	y_1		$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$		
		$\nabla y_2 = y_2 - y_1$		$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$	
x_2	y_2		$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$		$\nabla^4 y_0 = \nabla^3 y_4 - \nabla^3 y_3$
		$\nabla y_3 = y_3 - y_2$		$\nabla^3 y_4 = \nabla^2 y_4 - \nabla^2 y_3$	
x_3	y_3		$\nabla^2 y_4 = \nabla y_4 - \nabla y_3$		
		$\nabla y_4 = y_4 - y_3$			
x_4	y_4				

Example: Find the value of $f(y)$ at $x = 19$ for the data given below:

X_i	0	5	10	15	20	25
y_i	7	11	14	18	24	32

Solution: $n = 5$, $h = 5$, $K = \dfrac{x - 25}{5}$

for $x = 19 \Rightarrow K = \dfrac{19 - 25}{5} = -1.2$

$$P_5(x) = y_5 + \frac{\nabla y_5}{1!} K + \frac{\nabla^2 y_5}{2!} K(K+1) + \frac{\nabla^3 y_5}{3} K(K+1)(K+2)$$
$$+ \frac{\nabla^4 y_5}{4!} K(K+1)(K+2)(K+3) + \frac{\nabla^5 y_5}{5!} K(K+1)(K+2)(K+3)(K+4)$$

Difference table:

x_i	y_i	∇y_i	$\nabla^2 y_i$	$\nabla^3 y_i$	$\nabla^4 y_i$	$\nabla^5 y_i$
0	7					
5	11	4				
10	14	3	-1			
15	18	4	1	2		
20	24	6	2	1	-1	
25	32	8	2	0	-1	0
	y_5	∇y_5	$\nabla^2 y_5$	$\nabla^3 y_5$	$\nabla^4 y_5$	$\nabla^5 y_5$

$\therefore P_5(19) = 32 + 8(-1.2) + (-1.2)(-0.2) + \dfrac{(-1)}{24}(-1.2)(-0.2)(0.8)(1.8)$

$\therefore P_5(19) = 22.625$

$\therefore$ The interpolated point (x, y) is:

$X = 19$, $y = 22.625$

Exercises :-

1. Use NDDIP to find y at $x = 2.3$ from the following data:-

x	2	4	6	8	10
y	2	1	3	8	20

2. Find the value of (y) at $x = 2.5$ for the following data:

x	0	1	2	3
y	0	1	8	135