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INDEX OF REFRACTION BY INTERFERENCE METHODS

If a thickness *t* of a substance having an index of refraction *n* is introduced into the path of one of the interfering beams in the interferometer, the optical path in this beam is increased because of the fact that light travels more slowly in the substance and consequently has a shorter wavelength. The optical path is now *nt* through the medium, whereas it was practically *t* through the corresponding thickness of air ($n = 1$). Thus the increase in optical path due to insertion of the substance is
$(n - l)t$. This will introduce $(n - 1)t/\lambda)$. extra waves in the path of one beam; so if we call Δm the number of fringes by which the fringe system is displaced when the substance is placed in the beam, we have:

$$(n - 1)t = (\Delta m)\lambda \qquad 17 \quad)$$

In principle a measurement of Δm, *t*, and), thus gives a determination of *n*.
In practice, the insertion of a plate of glass in one of the beams produces a is continuous shift of the fringes so that the number Δm cannot be counted. With monochromatic fringes it is impossible to tell which fringe in the displaced set corresponds to one in the original set. With white light, the displacement in the fringes of different colors is very different because of the variation of *n* with wavelength, and the fringes disappear entirely. This illustrates the necessity of the compensating plate G_2 in Michelson's interferometer if white-light fringes are to be observed. If the plate of glass is very thin, these fringes may still be visible, and this affords a method of measuring *n* for very thin films. For thicker pieces, a practicable method is to use two plates of identical thickness, one in each beam, and to turn one gradually about a vertical axis, counting the number of monochromatic fringes for a given angle of rotation. This angle then corresponds to a certain known increase in effective thickness.
For the measurement of the index of refraction of gases, which can be introduced gradually into the light path by allowing the gas to flow into an evacuated tube, the interference method is the most practicable one. Several forms of refractometers have been devised especially for this purpose, of which we shall describe three, the Jamin, the Mach-Zehnder, and the Rayleigh refractometers.
Jamin's refractometer is shown schematically in Fig. 11(a). Monochromatic

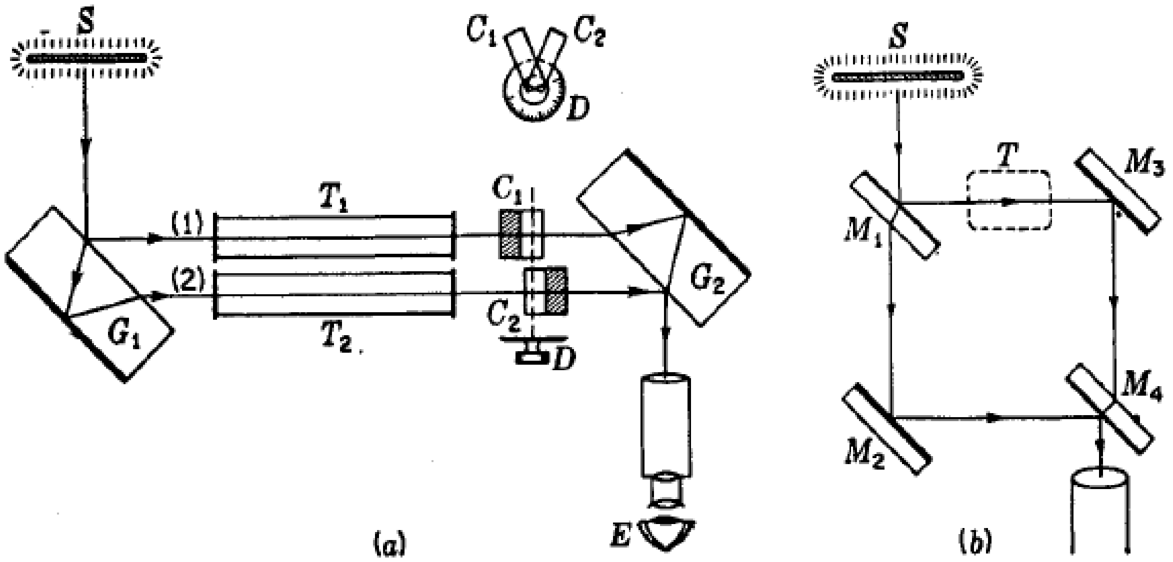


FIGURE 11
(a) The Jamin and (b) the Mach-Zehnder interferometer.

light from a broad source S is broken into two parallel beams 1 and 2 by reflection at the two parallel faces of a thick plate of glass G_1 .These two rays pass through to another identical plate of glass G_2 to recombine after reflection, forming interference fringes known as Brewster's If now the plates are parallel, the light paths will be identical. Suppose as an experiment we wish to measure the index of refraction of a certain gas at different temperatures and pressures. Two similar evacuated tubes T_1 and T_2 of equal length are placed in the two parallel beams.

Gas is slowly admitted to tube T_2. If the number of fringes Δm crossing the field is counted while the gas reaches the desired pressure and temperature, the value of *n* can be found by applying $(n - 1)t = (\Delta m)\lambda$

It is found experimentally that at a given temperature the value *n* - 1 is directly proportional to the pressure. This is a special case of the *Lorenz-Lorentz* law,* according to which:

$$\frac{n^2 - 1}{n^2 + 2} = (n - 1)\frac{n + 1}{n^2 + 2} = \text{const} \times \rho \qquad 18$$

Here *p* is the density of the gas. When *n* is very nearly unity, The interferometer devised by Mach and Zehnder, and shown in Fig. *11(b),* has a similar arrangement of light paths, but they may be much farther apart. The role of the two glass blocks in the Jamin instrument is here taken by two pairs of mirrors, the pair M_1 and M_2 functioning like G_1, and the pair *M3* and *M4* like G_2•

The first surface of M_1 and the second surface of M_4 are half-silvered. Although it is

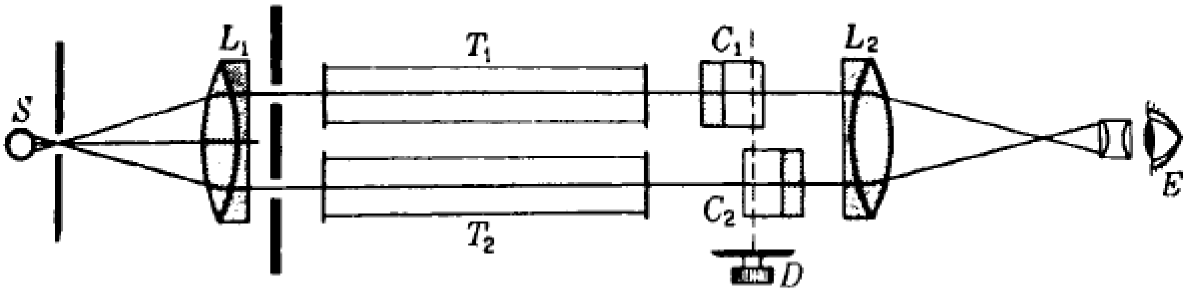


FIGURE 12
Rayleigh's refractometer.

more difficult to adjust, the Mach-Zehnder interferometer is suitable only for studying slight changes of refractive index over a considerable area and is used, for example, in measuring the flow patterns in wind tunnels Contrary to the situation in the Michelson interferometer, the light traverses a region such as *T* in the figure in only one direction, a fact which simplifies the study of local changes of optical path in that region.

The purpose of the compensating plates C1 and *C2* in Figs. 11(a) and 12 is to speed up the measurement of refractive index. As the two plates, of equal thickness, are rotated together by the single knob attached to the dial *D,* one light path is shortened and the other lengthened. The device can therefore compensate for the path difference in the two tubes. The dial, if previously calibrated by counting fringes, can be made to read the index of refraction directly. The sensitivity of this device can be varied at will, a high sensitivity being obtained when the angle between the two plates is small and a low sensitivity when the angle is large.

In Rayleigh's* refractometer (Fig. 13) monochromatic light from a linear source S is made parallel by a lens L_1 and split into two beams by a fairly wide double slit. After passing through two exactly similar tubes and the compensating plates, these are brought to interfere by the lens *Lz.* This form of refractometer is often used to measure slight differences in refractive index of liquids and solutions.

REFLECTION FROM A PLANE-PARALLEL FILM

Let a ray of light from a source S be incident on the surface of such a film at *A* (Fig. 13). Part of this will be reflected as ray 1 and part refracted in the direction *AF.*

Upon arrival at *F,* part of the latter will be reflected to *B* and part refracted toward *H.* At *B* the ray *FB* will be again divided. A continuation of this process yields two sets of parallel rays, one on each side of the film. In each of

these sets, of course, the intensity decreases rapidly from one ray to the next. If the set of parallel reflected rays is now collected by a lens and focused at the point *P,* each ray will have traveled
a different distance, and the phase relations may be such as to produce destructive or constructive interference at that point. It is such interference that produces the

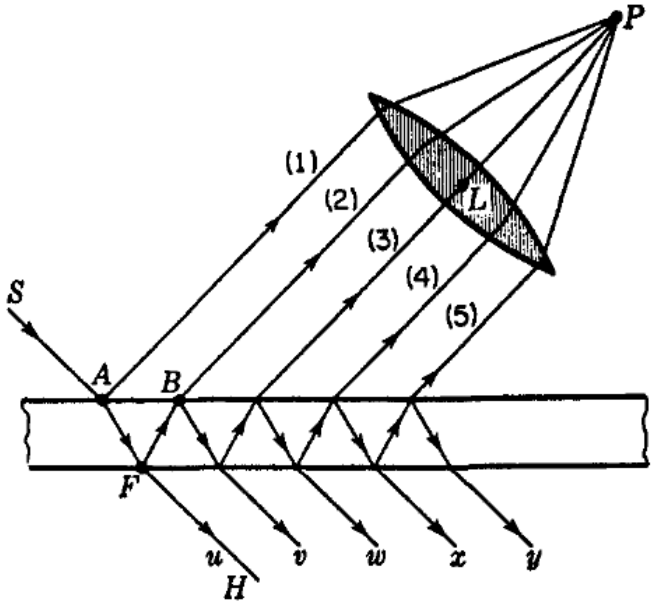


FIGURE 13
Multiple reflections in a plane-parallel film.

colors of thin films when they are viewed by the naked eye. In such a case *L* is the lens of the eye, and *P* lies on the retina.
In order to find the phase difference between these rays, we must first evaluate the difference in the optical path traversed by a pair of successive rays, such as rays 1 and 2.
If this path difference is a whole number of wavelengths, we might expect rays 1 and 2 to arrive at the focus of the lens in phase with each other and produce a maximum of intensity. However, we must take account of the fact that ray 1 undergoes a phase change of 1t at reflection, while ray 2 does not, since it is internally reflected. The condition

- $$2nd \cos \phi' = m\lambda \qquad \textit{Minima} \qquad (14f)$$

then becomes a condition for *destructive* interference as far as rays 1 and 2 are concerned.
As before, *m* = 0, 1, 2, ... is the order of interference.

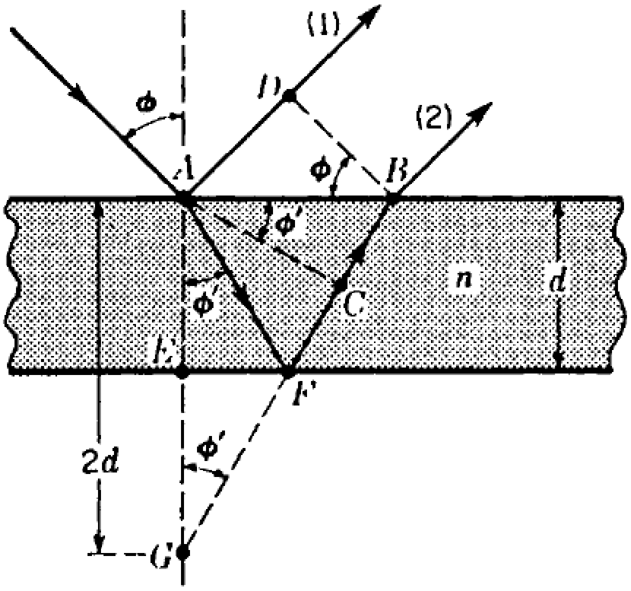


FIGURE 14C
Optical-path difference between two consecutive rays in multiple reflection (see Fig. 14A).

. On the other hand, if conditions are such that

- $$2nd \cos \phi' = (m + \tfrac{1}{2})\lambda \qquad \textit{Maxima} \qquad (14g)$$

ray 2 will be in phase with 1, but 3, 5, 7, . .. will be out of phase with 2, 4, 6, ...
we have taken the fraction reflected internally and externally to be the same. Adding the amplitudes of all the reflected rays but the first on the upper side of the film, we obtain the resultant amplitude,

$$A = atrt' + atr^3t' + atr^5t' + atr^7t' + \cdots$$
$$= atrt'(1 + r^2 + r^4 + r^6 + \cdots)$$

Since r is necessarily less than 1, the geometrical series in parentheses has a finite sum equal to 1/(1 - $r2$), giving

$$A = atrt' \frac{1}{1 - r^2}$$

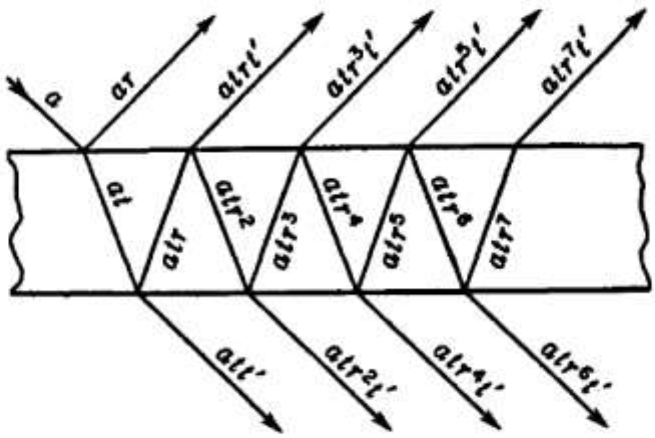


FIGURE 14D
Amplitudes of successive rays in multiple reflection.

But from Stokes' treatment,), $tt' = 1 - r2$; so we obtain finally

$A = ar$ (17)

This is just equal to the amplitude of the first reflected ray, so we conclude.

FRINGES OF EQUAL INCLINATION

If the image of an *extended* source reflected in a thin plane-parallel film is examined, it will be found to be crossed by a system of distinct interference fringes, provided the source emits monochromatic light and provided the film is sufficiently thin.

Each bright fringe corresponds to a particular path difference giving an integral value of min Eq. (l4g). For any fringe, the value of φ is fixed; so the fringe will have the form of the arc of a circle whose center is at the foot of the perpendicular drawn from the eye to the plane of the film. Evidently we are here concerned with fringes of equal inclination, and the equation for the path difference has the same form as for the circular fringes in the Michelson interferometer .

Note that if m is the order of interference for light incident on the film at φ = 0°, gives

$$m = \frac{2nd}{\lambda}$$

which would be a dark fringe. Since the path difference for the first, second, and third, etc., bright fringes will be at progressively larger angles of φ and φ' [Eq. (14g)], the successive path differences, *2nd* cos ', will be successively shorter and bright-light fringes will be at angles where *2nd* cos φ' is equal $(m – 1/2)\lambda.$, $(m - /2)\lambda.$, $(m –/2)\lambda.$, etc.

The necessity of using an extended source will become clear upon consideration of Fig. 14B. If a very distant point source S is used, the parallel rays will necessarily reach the eye at only one angle (that required by the law of reflection) and will be focused to a point *P.* Thus only one point will be seen, either bright or dark, according to the phase difference at this particular angle. It is true that if the source is not very far away, its image on the retina will be slightly blurred, because the eye must be focused for parallel rays to observe the interference. The area illuminated is small, however, and in order to see an extended system of fringes, we must obviously have many points S, spread out in a broad source so that the light reaches the eye from various directions.

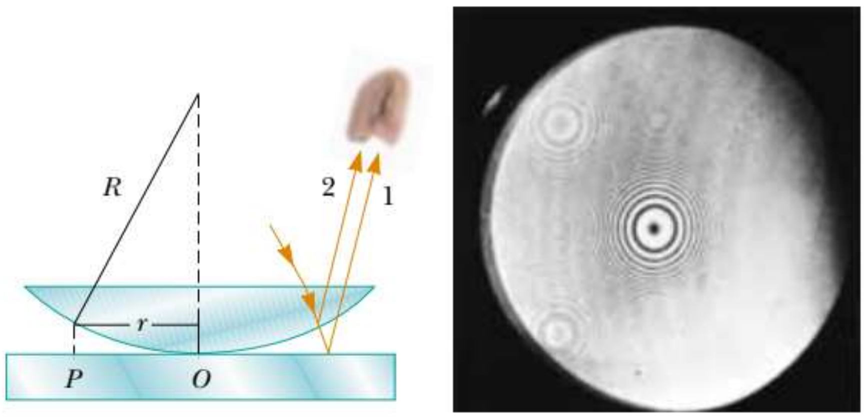
These fringes are seen by the eye only if the film is very thin, unless the light is reflected practically normal to the film. At other angles, since the pupil of the eye has a small aperture, increasing the thickness of the film will cause the reflected rays to get so far apart that only one enters the eye at a time. Obviously no interference can occur under these conditions. Using a telescope of large aperture, the lens may include enough rays for the fringes to be visible with thick plates, but unless viewed nearly normal to the plate, they will be so finely spaced as to be invisible. The fringes seen

$$2nt = m\lambda \qquad m = 0, 1, 2, \ldots$$

Newton's Rings

Another method for observing interference in light waves is to place a plano convex lens on top of a flat glass surface, as shown in Figure With this arrangement, the air film between the glass surfaces varies in thickness from zero at the point of contact to some value *t* at point *P*. If the radius of curvature *R* of the lens is much greater than the distance *r*, and if the system is viewed from above using light of a single wavelength , a pattern of light and dark rings is observed, as shown in Figure b. These circular fringes, discovered by Newton, are called **Newton's rings.**
The interference effect is due to the combination of ray 1, reflected from the flat plate, with ray 2, reflected from the curved surface of the lens. Ray 1 undergoes a phase change of 180° upon reflection (because it is reflected from a medium of higher refractive index),

discovered by Newton, are called **Newton's rings.**



(a) (b)

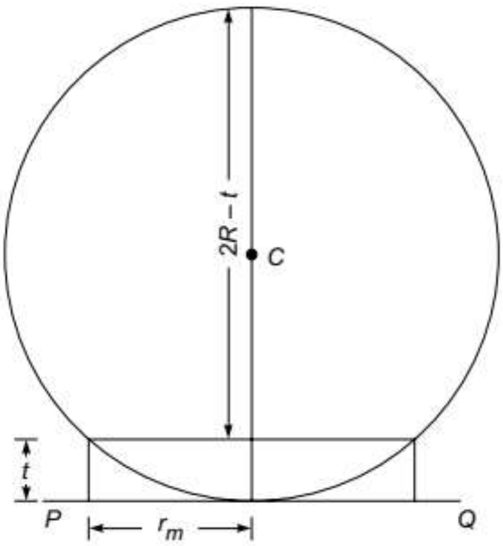


Figure 15 : Newton's rings

r_m represents the radius of the mth dark ring;the thickness of the air film (where the mth dark ring is formed) is t.
the optical path difference between the two waves is very nearly equal to 2*nt*, where *n* is the refractive index of the film and *t* is the thickness of the film. Thus, whenever the thickness of the air film satisfies the condition

$$r_m^2 \approx 2Rt$$

$$2t \approx \frac{r_m^2}{R}$$

If a liquid of refractive index *n* is introduced between the lens and the glass plate, the radii of the dark rings are given by

$$r_m = \sqrt{\frac{m\lambda R}{n}}$$

(because it is reflected from a medium of lower refractive index). Hence,

the optical path difference between the two waves is very nearly equal to 2*nt*, where *n* is the refractive index of the film and *t* is the thickness of the film. Thus, whenever the thickness of the air film satisfies the condition

$R^2 = r_m^2 + (R - t)^2$

r_m^2=2Rt - t^2

$R > t$

2Rt >>t^2

condition of darkness at Q is that:

2t=mλ

r_m^2 = mλR

$r_m = \sqrt{m\lambda R}$

Diameter D_m=2 r_m

$D_m = 2\sqrt{m\lambda R}$ since m=1,2,3………

$D_m = \sqrt{4m\lambda R}$

The Fabry-Perot interferometer

This instrument uses multiple beam interference by division of amplitude. Figure 16 shows a beam from a point on an extended source incident on two reflecting surfaces separated by a distance *d*. Note that this distance is the optical distance i.e. the product of refractive index *n* and physical length. For convenience we will omit *n* from the equations that follow but it needs to be included when the space between the reflectors is not a vacuum. An instrument with a fixed *d* is called an *etalon*. Multiple beams are generated by partial reflection at each surface resulting in a set of parallel beams having a relative phase shift δ introduced by the extra path 2*d*cosθ between

successive reflections which depends on the angle θ of the beams relative to the axis. Interference therefore occurs at infinity - the fringes are of equal inclination and localized at infinity. In practice a lens is used and the fringes observed in the focal plane where they appear as a pattern of concentric circular rings

The Fabry-Perot interference pattern

This is done in all the text books (consult for details). The basic idea is as follows:

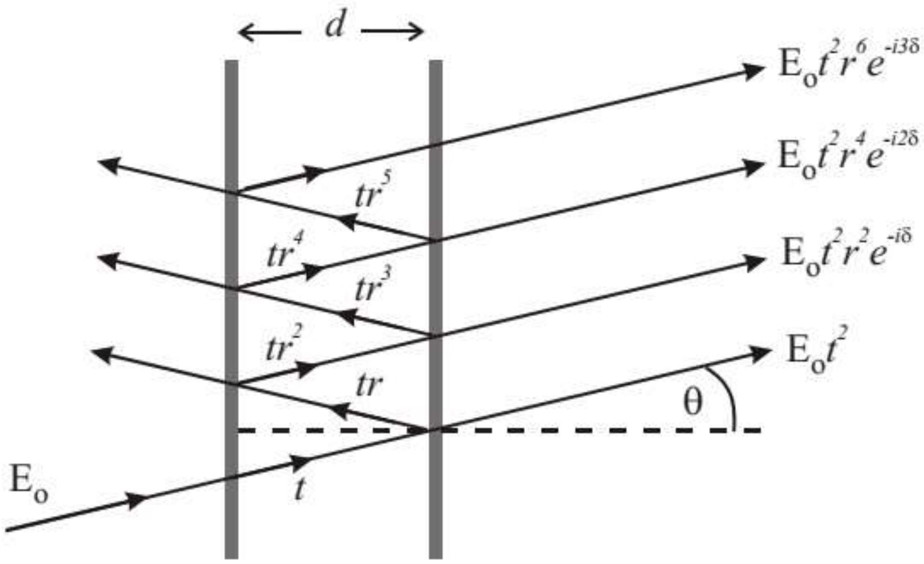


Figure 16. Multiple beam interference of beams reflected and transmitted by parallel surfaces with amplitude reflection and transmission coefficients r_i, t_i respectively.
Amplitude reflection and transmission coefficients for the surfaces are r_1, t_1 and r_2, t_2, respectively.
The phase difference between successive beams is:

$$\delta = \frac{2\pi}{\lambda} 2d\cos\theta$$

An incident wave $E_0e^{-i\omega t}$ is transmitted as a sum of waves with amplitude and phase given by:

$$E_t = E_0t_1t_2e^{-i\omega t} + E_0t_1t_2r_1r_2e^{-i(\omega t-\delta)} + E_0t_1t_2r_1^2r_2^2e^{-i(\omega t-2\delta)} + \ldots etc.$$

Taking the sum of this Geometric Progression in $r_1r_2e^{i\delta}$

$$E_t = E_0 t_1 t_2 e^{i\omega t}\left[\frac{1}{1 - r_1 r_2 e^{i\delta}}\right]$$

and multiplying by the complex conjugate to find the transmitted Intensity:

$$I_t = E_t E_t^* = E_0^2 t_1^2 t_2^2\left[\frac{1}{1 + r_1^2 r_2^2 - 2 r_1 r_2 \cos\delta}\right]$$

writing $E_0^2 = I_0$, $r_1 r_2 = R$ and $t_1 t_2 = T,$ and $\cos\delta = (1 - 2\sin^2\delta/2)$:

$$I_t = I_0 \frac{T^2}{(1-R)^2}\left[\frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2\delta/2}\right]$$

If there is no absorption in the reflecting surfaces $T = (1 - R)$ then defining

$$\frac{4R}{(1-R)^2} = \Phi$$

$$I_t = I_0\left[\frac{1}{1 + \Phi\sin^2\delta/2}\right]$$

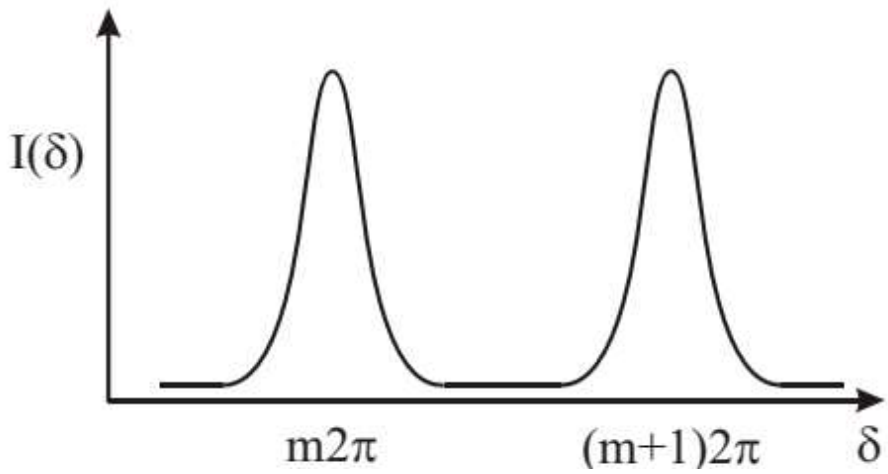


fig .17 Observing Fabry-Perot fringes

The Airy function describes the shape of the interference fringes. Figure 17 shows the intensity as a function of phase shift δ. The fringes occur each time δ is a multiple of

2π , *m* is an integer, the order of the fringe.

$$\delta = \frac{2\pi}{\lambda} 2d\cos\theta = m2\pi$$

The fringes of the Airy pattern may be observed by a system to vary *d*, λ, or θ. A system for viewing many whole fringes is shown in figure 17. An extended source of monochromatic light is used with a lens to form the fringes on a screen. Light from any point on the source passes through the F.P. at a range of angles illuminating a number of fringes. The fringe pattern is formed in the focal plane of the lens.

Extended

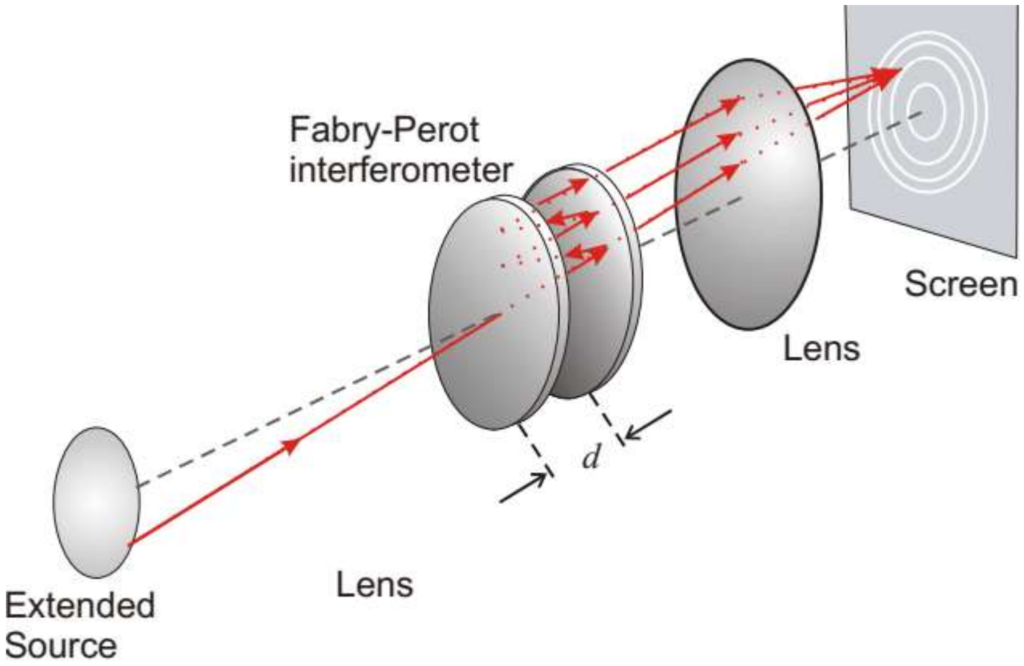


Figure 18 Schematic diagram of arrangement to view Fabry-Perot fringes. Parallel light from the Fabry-Perot is focussed on the screen.

PROBLEMS

8.1 Young's experiment is performed with orange light from a krypton arc. If the fringes are measured with a micrometer eyepiece at a distance 100 cm from the double slit, it is found that 25 of them occupy a distance of 12.87 mm between centers. Find the distance between the centers of the two slits.

Ans. 1.1297 mm

8.2 A double slit with a separation of 0.250 mm between centers is illuminated with green light from a cadmium-arc lamp. How far behind the slits must one go to measure the fringe separation and find it to be 0.80 mm between centers?

8.3 When a thin film of transparent plastic is placed over one of the slits in Young's double-slit experiment, the central bright fringe, of the white-light fringe system, is displaced by 4.50 fringes. The refractive index of the material is 1.480, and the effective wavelength of the light is 5500 A. *(a)* By how much does the film increase the optical path? *(b)* What is the thickness of the film? *(c)* What would probably be observed if a piece of the material 1.0 mm thick were used? *(d)* Why?

8.4 L1oyd's-mirror experiment is readily demonstrated with microwaves, using as a reflector a sheet of metal lying flat on the table. If the source has a frequency of 12,000 MHz and is located 10.0 cm above the sheet-metal surface, find the height above the surface of the first two maxima 3.0 m from the source.

Ans. (a) 18.750 cm, (b) 56.25 cm

Note: A phase change of π: occurs upon reflection;.

8.5 A Fresnel biprism is to be constructed for use on an optical bench with the slit and the observing screen 180.0 cm apart. The biprism is to be 60.0 cm from the slit. Find the angle between the two refracting surfaces of the biprism if the glass has a refractive index n = 1.520, sodium yellow light is to be used, and the fringes are to be 1.0 mm apart.

8.6 A Fresnel biprism of index 1.7320 and with apex angles of 0.850° is used to form interference fringes. Find the fringe separation for red light of wavelength 6563 A when the distance between the slit and the prism is 25.0 cm and that between the prism and the screen is 75.0 cm.

8.7 What must be the angle in degrees between the two Fresnel mirrors in order to produce sodium light fringes 1.0 mm apart if the slit is 40.0 cm from the mirror intersection and the screen is 150.0 cm from the slit? Assume λ= 5.893×10^{-5} cm.

Ans. 0.06331°

8.8 How far must the movable mirror of a Michelson interferometer be displaced for 2500 fringes of the red cadmium line to cross the center of the field of view?
8.9 If the mirror of a Michelson interferometer is moved 1.0 mm, how many fringes of the blue cadmium line will be counted crossing the field of view?

8.10 Find the angular radius of the tenth bright fringe in a Michelson interferometer when the central-path difference *(2d)* is *(a)* 1.50 mm and *(b)* 1.5 cm. Assume the orange light of a krypton arc is used and that the interferometer is adjusted in each case so
that the first bright fringe forms a maximum at the center of the pattern.
Ans. (a) 4.885°, *(b) 1.542°*