Chapter -2-

Solving of non-linear equations (Root finding)

General notes :-

- Our equations must be written in the form:

$$f(x)=0$$

for example:

$$x^2-3x+2=0$$

$$xe^x-4=\ln x \Rightarrow xe^x-\ln x-4=0$$

f(x) is any function of the variable(x).

The root of the equation $f(x)=0$ is the value of f(x) hich satisfies the equation, or, the root is the alue of (x) which makes f(x) equal to zero.

- We shall denote the root by $(\bar{x})$.

- The equation $f(x)=0$ may have more than one oot.

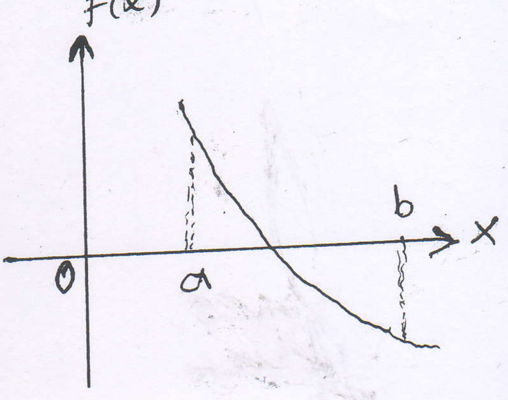
- We shall take some iterative numerical ethods for root finding.

- In many of these methods we need to find the nterval which contains the root (the root maybe ositive or negative).

- For a positive root (+ve)

we make the table :

x	0	0.5	1	1.5	2
f(x)	+	+	+	−	−
			a	b	



there is a root in the interval [a,b].

- For anegative root we take (-ve) values for x) (starting from Zero if possible.

- In iterative methods, we get closer and closer to the real root by calculating many values of (x).

- Each calculated x is known as an instantaneous root: $x_1, x_2, x_3, \cdots$ (in general x_i) (i may take 0-value

- The more close x_i to the real root, the more accurate the solution.

- We may use the notations: x_i, x_{i+1}, x_{i-1}.

  for example, if $i=3$:

  $x_i = x_3$, $x_{i+1} = x_4$, $x_{i-1} = x_2$.

- When the instantaneous root (x_i) gets closer to the real root, the function $f(x_i)$ gets closer to Zero.

or, when $x_i \rightarrow \bar{x}$ then $f(x_i) \longrightarrow 0$.

- In general, full accuracy is not obtained in umerical methods and we may consider the root as that value of (x_i) which makes $|f(x_i)| \leq \epsilon$, where ϵ is a small quantity.

- For smaller ϵ we get higher accuracy.

Accuracy maybe represented by different stopping Conditions :-

* Absolute error : E_{abs} : $|X_{i+1} - X_i| \le \epsilon$,

* Relative error : E_{rel} : $\left|\frac{x_{i+1} - X_i}{X_{i+1}}\right| \le \epsilon$,

* Percentage error : e% : $\left|\frac{x_{i+1} - X_i}{X_{i+1}}\right| * 100 \le \epsilon$

* If $|f(x_i)| \le \epsilon$

or $|f(x_{i+1})| \le \epsilon$

* Coincidence in decimal digits for X_i - Values (or X_{i+1} - Values)

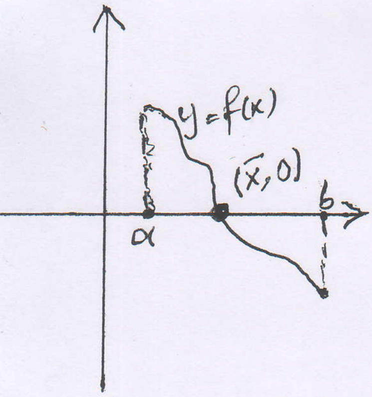
Correct to 2D

Correct to 3D .

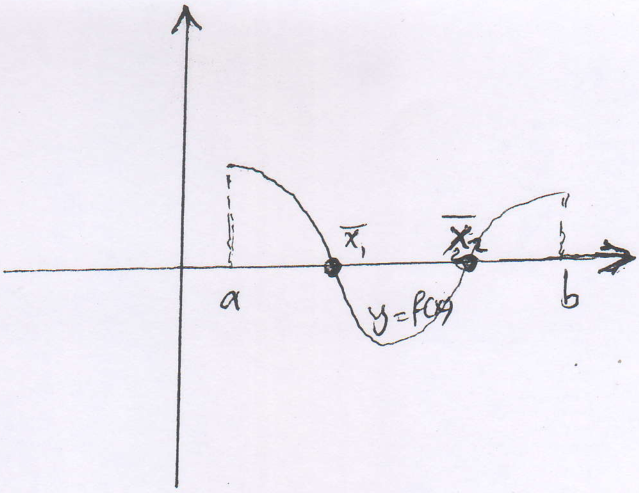
here are two methods to find the initial value
the root for the equations :-

Graphical method :

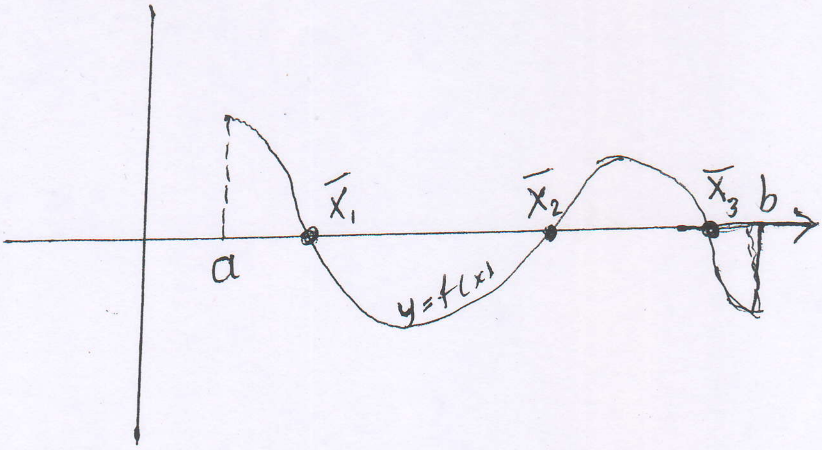
e roots of the equation $f(x)=0$ are the intersection
ints of the curve of the function $y=f(x)$ with
e x-axis.



One root

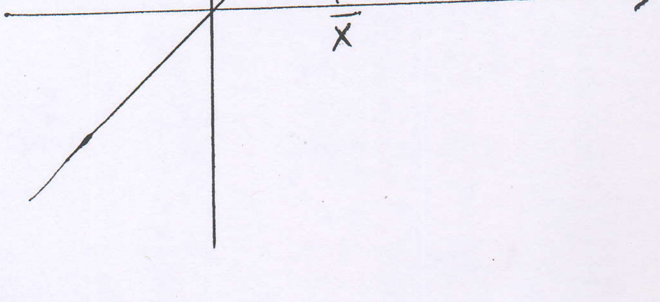


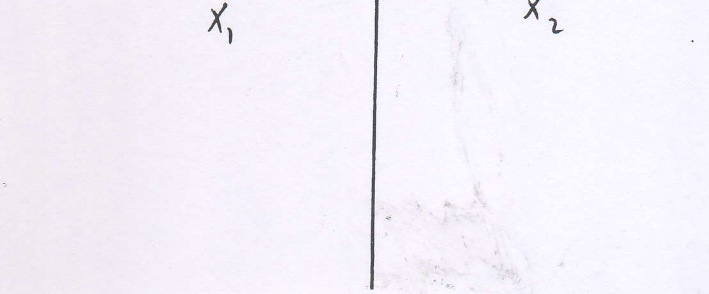
two roots



three roots

some cases it is be~~t~~ter to write an equation $f(x)=0$
n the form : $f_1(x) = f_2(x)$
fter that we draw two functions $y_1=f_1(x)$ and $y_2=f_2(x)$
e the intersection points $(\bar{x},\bar{y})$ of the curves then $\bar{x}$
epresents a root of the equation.





ample Find the solutions of the following equations by
e graphical method?

x^2-1 , ② xe^x-1

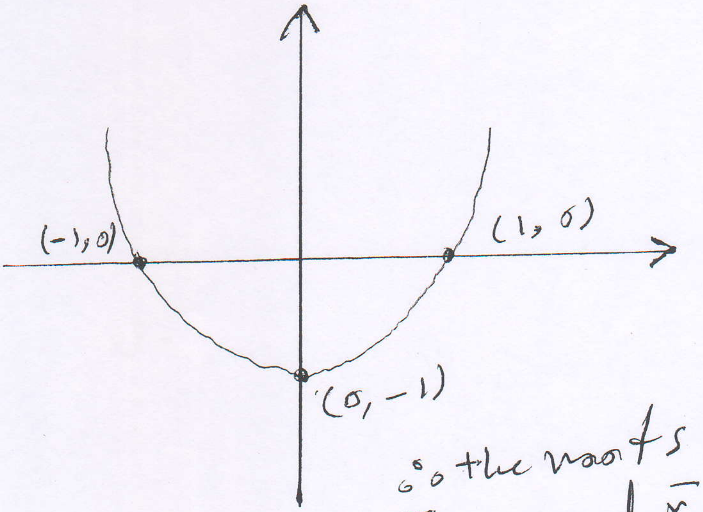
olution

case ⓐ

$y=x^2-1$

x	y
0	-1
-1	0
1	0

~~case ⓑ~~

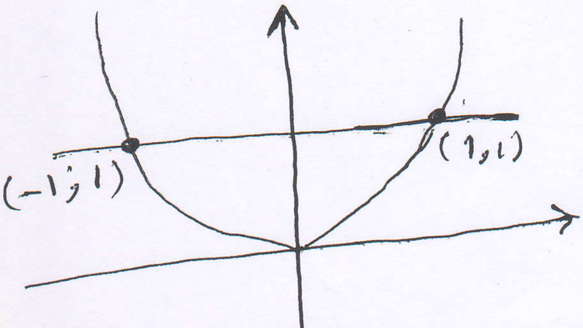


∴ the roots are
$\bar{x}=1$ and $\bar{x}=-1$

Case ⓑ

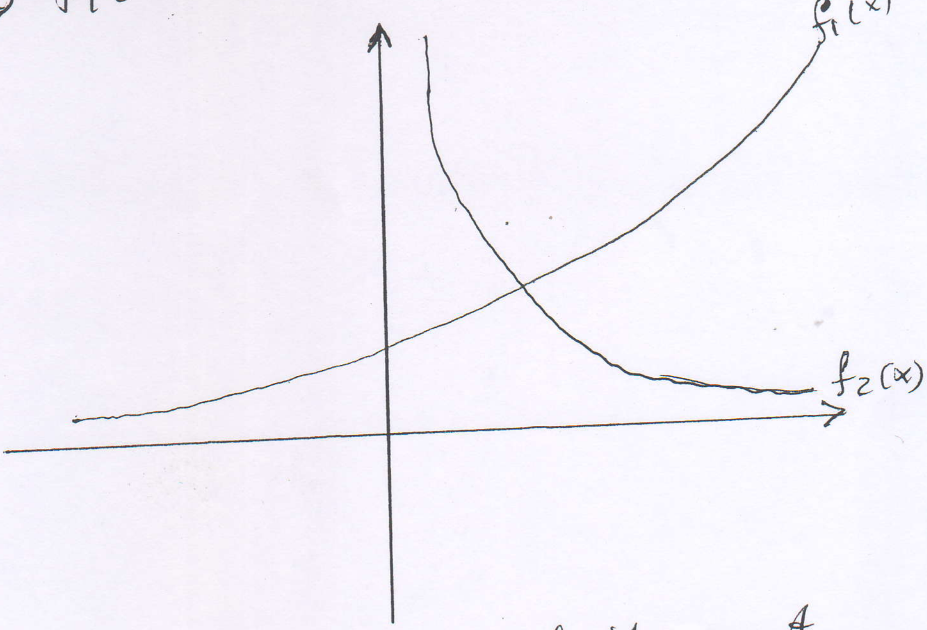
$x^2-1=0$

$\Rightarrow x^2=1 \quad \Rightarrow f_1(x)=x^2$ and $f_2(x)=1$



∴ The roots are
$\bar{X}=1$ and $\bar{X}=-1$

$x e^{x} = 1 \Rightarrow e^{x} = \frac{1}{x} \Rightarrow f_1(x) = e^{x}$ and $f_2(x) = \frac{1}{x}$



the location of the root on the interval (0,1)

Analytical method:

Analytical method based on the mean-Value heorem. Let $f(x)$ be a real continuous function on he interval $[a,b]$, where a and b real numbers uch that $a<b$, if $f(a)$ and $f(b)$ different n signs then there exists at least one real root on the interval $[a,b]$.

- The accuracy to determine the location of roots depend on the divided of the interval $[a,b]$ into Subintervals.

- The number of positive roots of $f(x)$ is the number of change in signs of $f(x)$.

The number of negative roots of $f(x)$ is the number of change in signs of $f(-x)$.

xample: Find the location of the roots of the following equation by analytical method on the interval $[-1, 1]$?

$f(x) = x^2 - x - 1$

Solution

$f(x) = +x^2 - x - 1$

∴ there is One +ve root.

$f(-x) = x^2 + x - 1$

∴ there is One −ve root

x	-1	1
	$+$	$-$

xample: Find the locations of roots of the following equations by analytical method:

① $f(x) = x^2 - x - 1$ in $I = [-2, 2]$

f have positive and negative roots

x	-2	-1	0	1	2
$f(x)$	$+$	$+$	$-$	$-$	$+$

∴ there are two roots in the $(-1, 0)$ and $(1, 2)$

) $f(x) = x^3 - 5x^2 + 2x + 8$, $I = [-4, 4]$?

$f(x) = \underset{\curvearrowright}{x^3 - 5x^2} \underset{\curvearrowright}{+ 2x} + 8$

there are two +ve roots

$f(-x) = -x^3 - 5x^2 \underset{\curvearrowright}{- 2x + 8}$

there is one −ve root

x	-4	-3	-2	-1	0	1	2	3	4
(x)	−	−	−	0	+	+	0	−	0

Exercises:

) $f(x) = x^3 - x^2 - 2x + 1$, $I = [-4, 4]$?

) $f(x) = x^4 - 7x^3 - x^2 + 26x - 10$, $I = [-8, 8]$?