Sets and Subsets:

The concept of set is fundamental in modern mathematics. The sets, as they are usually conceived, have elements. The elements of a set may be anything, numbers, persons, mountains, rivers, --- ect.
We shall denote sets by capital letters $A, B, C, X, Y, \cdots$ ect. and their elements by small letters $a, b, c, x, y, \ldots$ ect.
We write $a \in A$ which is read "a belongs to A", when a is not a member of A, we write $a \notin A$.
For example, let the elements of the set A be a, e, i, o, u then we write $A = \{a, e, i, o, u\}$, this is called tabular form of the set.
A set may also be specified by stating properties which its elements must satisfy.
$A = \{x : x \text{ is a vowel in English alphabet}\}$.

Sets of Numbers:

$\mathbb{N}$ = the set of natural numbers $= \{0, 1, 2, 3, \ldots\}$
$\mathbb{Z}$ = the set of integer numbers $= \{0, \mp 1, \mp 2, \mp 3, \ldots\}$
$\mathbb{Q}$ = the set of rational numbers $= \{\frac{p}{q} : p, q \in \mathbb{Z},\ q \neq 0\}$
$\mathbb{R}$ = the set of real numbers.
$\mathbb{C}$ = the set of complex numbers

Cartesian Product of Sets:

The cartesian product of two sets A and B is the set
$A \times B = \{(a, b) : a \in A,\ b \in B\}$.

Example:

Let $A = \{1, 2\}$ and $B = \{3, 4, 5\}$, find $A \times B$?

Solution:

$$A \times B = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)\}.$$

Relations:

A relation from a set A to set B is a subset of $A \times B$ and is denoted by R.
The domain of a relation R is the set of all first coordinates of the members of R. The range of R is the set of all second coordinates of the members of R. Thus:

$$\text{Dom}\, R = \{a : (a, b) \in R \text{ for some } b \in B\}$$
$$\text{Ran}\, R = \{b : (a, b) \in R \text{ for some } a \in A\}$$

Example:

Let $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$
$R = \{(1,2), (2,3), (3,4)\}$, find $\text{Dom}\, R$ and $\text{Ran}\, R$?

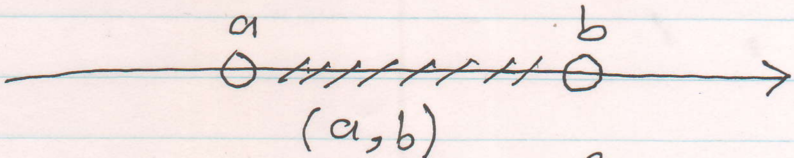
Solution:

$$\text{Dom}\, R = \{1, 2, 3\}$$

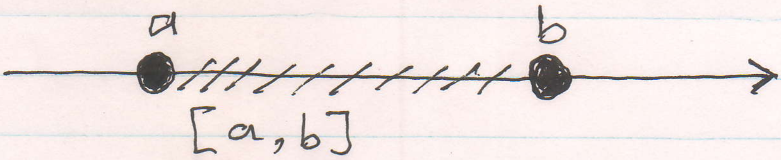
$$\text{Ran}\, R = \{2, 3, 4\}$$

Intervals:

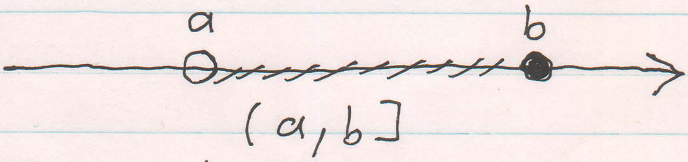
An open interval on $\mathbb{R}$ (the set of real numbers) is any set of the form $\{x \in \mathbb{R} : a < x < b\}$ and is denoted by (a, b).

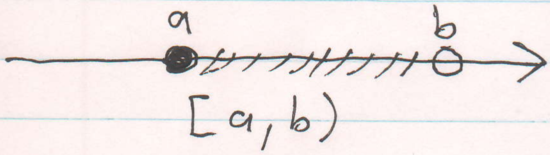


A closed interval is any set of the form $\{x \in \mathbb{R} : a \leq x \leq b\}$ and is denoted by $[a, b]$.



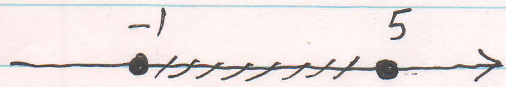
Similarly the sets $\{x \in \mathbb{R} : a \leq x < b\}$ and $\{x \in \mathbb{R} : a < x \leq b\}$ are called right half open and left half open intervals and are denoted by $[a, b)$ and $(a, b]$ respectively:



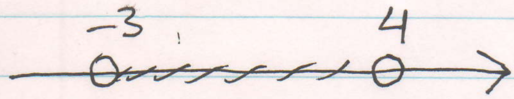


Example:

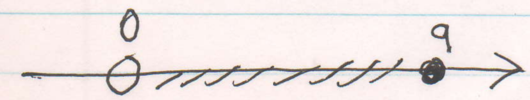
$\{x \in \mathbb{R} : -1 \leq x \leq 5\} = [-1, 5]$



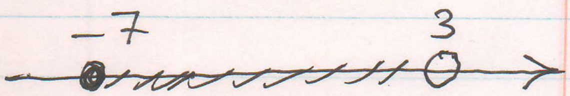
$\{x \in \mathbb{R} : -3 < x < 4\} = (-3, 4)$



$\{x \in \mathbb{R} : 0 < x \leq 9\} = (0, 9]$



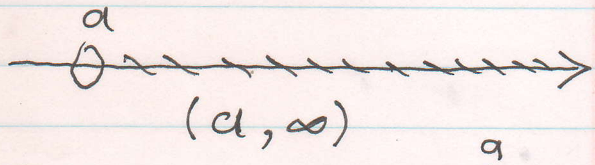
$\{x \in \mathbb{R} : -7 \leq x < 3\} = [-7, 3)$



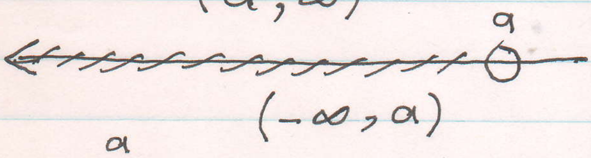
Infinite Intervals:

The infinite intervals defined by:

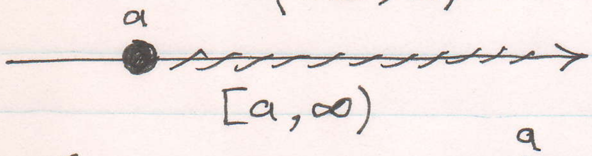
$\{x \in \mathbb{R} : x > a\} = (a, \infty)$



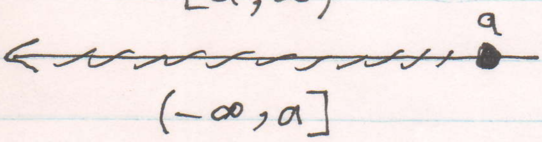
$\{x \in \mathbb{R} : x < a\} = (-\infty, a)$



$\{x \in \mathbb{R} : x \geqslant a\} = [a, \infty)$



$\{x \in \mathbb{R} : x \leqslant a\} = (-\infty, a]$



Inequalities:

Some basic properties of inequalities:
If a, b and c are any given real numbers, then
1. either $a > b$, $a = b$ or $a < b$.
2. if $a > b$ and $b > c$ then $a > c$.
3. if $a > b$ then $a + c > b + c$.
4. if $a > b$ and $c > 0$ then $ac > bc$.
5. if $a > b$ and $c < 0$ then $ac < bc$.

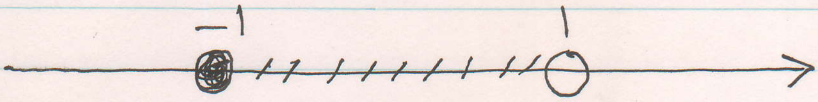
Example: Solve $1 \leqslant 2x + 3 < 5$, sketch the solution set on the number line and write it in terms of intervals.

Solution:

$$1 \leqslant 2x + 3 < 5$$
$$-2 \leqslant 2x < 2$$

$-1 \leq x < 1$

the solution set is $[-1, 1)$



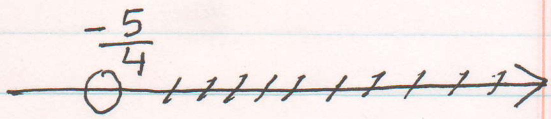
Example:

Solve $10x + 4 > 2(x-3)$, sketch the Solution set on the number line, and write it in terms of intervals.

Solution:

$$10x + 4 > 2(x-3)$$
$$10x + 4 > 2x - 6$$
$$10x - 2x > -6 - 4$$
$$8x > -10$$
$$x > -\frac{10}{8}$$
$$x > -\frac{5}{4}$$

the solution set is $\left(-\frac{5}{4}, \infty\right)$



Example: Solve $x + 5 < 9x + 9$, sketch the Solution set on the number line, and write it in terms of intervals.

Solution:

$$x + 5 < 9x + 9$$
$$x - 9x < 9 - 5$$
$$-8x < 4$$
$$x > -\frac{4}{8}$$
$$x > -\frac{1}{2}$$

the solution set is $\left(-\frac{1}{2}, \infty\right)$