Functions:

Functions and their graphs:

Definition: A function f (or a mapping f) from a set A to a set B is a rule that assigns to each element a of A exactly one element b of B. The set A is called the domain of f and the set B is called the codomain of f. If f assigns b to a, then b is called the image of a under f. The subset of B comprised of all the images of elements of A under f (which is denoted by $f(A)$) is called the image of A under f (or the range of f).
We use $f: A \longrightarrow B$ to mean that f is a function from A to B. We will write $f(a) = b$ to indicate that b is the image of a under f.

Example: Let $A = \{2, 4, 5\}$, $B = \{1, 2, 3, 6\}$ and $f: A \to B$ be the function defined by $f(2) = 1$, $f(4) = 3$, $f(5) = 6$. Then the domain of f is $A = \{2, 4, 5\}$ the codomain of f is $B = \{1, 2, 3, 6\}$, and the range of $f = \{1, 3, 6\}$.

Counter example:

Let $C = \{1, 2, 3, 4\}$ and $D = \{2, 3, 4, 5\}$, and let h be the rule defined by $h(1) = 2$, $h(1) = 4$, $h(2) = 3$ $h(3) = 5$, $h(4) = 4$, then h is not a function from C to D since there are two different elements

(2 and 4) belong to D are assigned to the same element 1 of C.

Example: Find the domain and the range of the function f defined by $f(x) = \sqrt{x+10}$

Solution: For $y = f(x) = \sqrt{x+10}$ to be real, $x+10$ must be greater than or equal to 0. That is, $x+10 \geq 0$ which means that $x \geq -10$. Thus the domain is $\{x : x \geq -10\}$ and the range is $\{y : y \geq 0\}$.

Exercises:

1) Let $A = \{2, 4, 5, 7\}$, $B = \{1, 2, 3, 6, 9\}$, and $f: A \to B$ be the function defined by $f(2) = 9$, $f(4) = 3$, $f(5) = 6$, $f(7) = 2$. Find the domain of f, the codomain of f, and the range of f?

2) Let f be a function defined by $f(x) = \frac{1}{x+2}$. Find the domain and the range of the function f?

3) Find the domain and the range of the function f defined by $f(x) = \sqrt{2x-9}$?

Definition: The graph of a function f is the line passing through all the points $(x, f(x))$ on the xy-plane

Definition: The y-coordinate of the point where a graph of a function intersect the y-axis is called the y-intercept

of the function.

Definition: The x-coordinate of a point where a graph of a function intersects the x-axis is called an x-intercept of the function.

Remarks:-

1) The graph of any function f has at most one y-intercept. The graph of the function f has exactly one y-intercept if 0 is in the domain of the function f and the y-intercept is $f(0)$.

2) The graph of any function f has no x-intercept if there is no x in the domain of the function f such that $f(x) = 0$. The graph of a function f has one or more than one x-intercepts if $f(x) = 0$ for some x in the domain of f, and the number of x-intercepts is the number of the distinct solutions of the equation $f(x) = 0$.

Properties of functions:

1) A function $y = f(x)$ is called an even function of x if $f(-x) = f(x)$, $\forall x$.

2) A function $y = f(x)$ is called an odd function of x if $f(-x) = -f(x)$, $\forall x$.