Linear Functions and their Graphs :-

Definition: A function $f: \mathbb{R} \to \mathbb{R}$ is called a linear function if f is defined by:
$f(x) = ax + b$, $a \neq 0$, where a and b are real numbers.

Example: The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x + 12$ is a linear function.

Example: The function $g: \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x - 0.2$ is a linear function.

Example: The function $h: \mathbb{R} \to \mathbb{R}$ defined by $h(x) = -\frac{3}{2}x + 1$ is a linear function.

Example: Let $f: \mathbb{R} \to \mathbb{R}$ be the linear function defined by $f(x) = 4x + 10$. Find the x-intercept and the y-intercept of f.

Solution: $f(x) = 0 \Rightarrow 4x + 10 = 0 \Rightarrow 4x = -10$
$\Rightarrow x = -\frac{10}{4} = -2.5$

Therefore the x-intercept is -2.5
$f(0) = 10 \Rightarrow$ the y-intercept is 10.

Example: Let $g: \mathbb{R} \to \mathbb{R}$ be the linear function defined by $g(x) = \frac{1}{5}x - 6$. Find the x-intercept and the y-intercept of g.

Solution: $g(x)=0 \Rightarrow \frac{1}{5}x-6=0 \Rightarrow \frac{1}{5}x=6$
$\Rightarrow x=30$
Therefore the x-intercept is 30.
$g(0)=-6 \Rightarrow$ the y-intercept is -6.

Graph of a linear function:

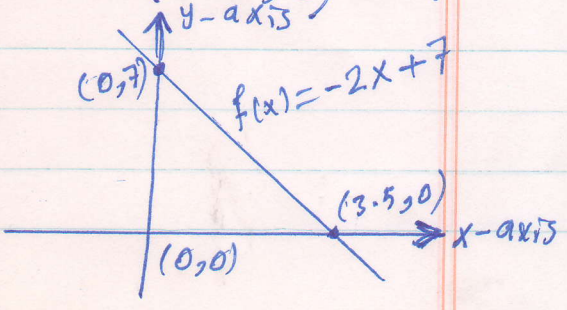
The graph of a linear function f is the straight line passing through the two points $(a,0)$ and $(0,b)$ where a is the x-intercept of the function f and b is the y-intercept of the function f.

Remark: The graph of any linear function f has exactly one x-intercept and has exactly one y-intercept.

Example: Let $f:\mathbb{R}\to\mathbb{R}$ be the linear function defined by $f(x)=-2x+7$. Find the x-intercept and the y-intercept of f then graph the function f.

Solution: $f(x)=0 \Rightarrow -2x+7=0 \Rightarrow -2x=-7$
$\Rightarrow x=\frac{-7}{-2}=3.5$

Therefore the x-intercept is 3.5.
$f(0)=7 \Rightarrow$ the y-intercept is 7.
Thus the graph of the function f is the straight line passing through the two points $(3.5,0)$ and $(0,7)$.
Thus the graph of the function f is the following graph



Example:

Let $g: \mathbb{R} \to \mathbb{R}$ be the linear function defined by $g(x) = 4x + 12$. Find the x-intercept and the y-intercept of g, then graph the function g.

Solution:- $g(x) = 0 \Rightarrow 4x + 12 = 0$

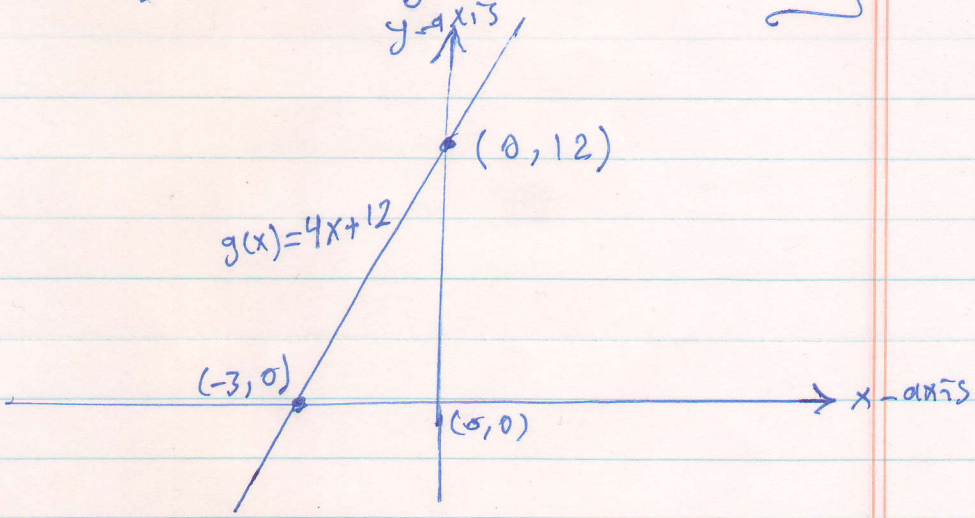
$\Rightarrow 4x = -12$

$\Rightarrow x = \frac{-12}{4} = -3$

Therefore the x-intercept is -3

$g(0) = 12 \Rightarrow$ the y-intercept is 12.

Thus the graph of the function g is the following graph



Exercises:

1) Let $f: \mathbb{R} \to \mathbb{R}$ be the linear function defined by $f(x) = 3x - 10$. Find the x-intercept and the y-intercept of f.

2) Let $g: \mathbb{R} \to \mathbb{R}$ be the linear function defined by $g(x) = 0.3x + 0.7$. Find the x-intercept and the y-intercept of g.

3) Let $f: \mathbb{R} \to \mathbb{R}$ be the linear function defined by $f(x) = -4x + 8$. Find the x-intercept and the y-intercept of f, then graph the function f.

4) Let $g: \mathbb{R} \to \mathbb{R}$ be the linear function defined by $g(x) = 5x + 15$. Find the x-intercept and the y-intercept of g, then graph the function g.