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5) $y = \sec x = \frac{1}{a} \qquad (a \neq 0)$

$= \sec(x \text{ radians}) = \sec(t \text{ degrees}) = \sec t^\circ$

6) $y = \csc x = \frac{1}{b} \qquad (b \neq 0)$

$= \csc(x \text{ radians}) = \csc(t \text{ degrees}) = \csc t^\circ$

Remark 1: Definition 1 uses the standard function notation , $y = f(x)$, with f replaced by the name of a particular trigonometric function . For example , $y = \cos x$ actually means $y = \cos(x)$ and $\cos t^\circ$ actually means $\cos(t^\circ)$.

Remark 2: Remember that $t^\circ = t \times \frac{\pi}{180}$ *radians* and

$x \text{ radians} = (x \times \frac{180}{\pi})^\circ$.

Theorem 1:

For any real number x we have the following triyonometric identities :

1) $\csc x = \frac{1}{\sin x}$.

2) $\sec x = \frac{1}{\cos x}$.

3) $\cot x = \frac{1}{\tan x}$.

4) $\tan x = \frac{\sin x}{\cos x}$.

5) $\cot x = \frac{\cos x}{\sin x}$.

6) $\sin(-x) = -\sin(x)$.

7) $\cos(-x) = \cos(x)$.

8) $\tan(-x) = -\tan(x)$.

9) $\cot(-x) = -\cot(x)$.

10) $\sin^2 x + \cos^2 x = 1$.

11) $\sec^2 x = \tan^2 x + 1$.

12) $\csc^2 x = \cot^2 x + 1$.

S 2.5: Graphs of Sine and Cosine Functions

2.5.1: Table for values of $\sin x$, $\cos x$, and $\tan x$ for selected values of x

Values of x						
	Degrees	0	30	45	60	90
	Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$		0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$		1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan x$		0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Values of x						
	Degrees	120	135	150	180	270
	Radians	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$
$\sin x$		$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1
$\cos x$		$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	0
$\tan x$		$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	Undefined

Definition: A function f is periodic if there exists a positive real number p such that $f(x) = f(x+p)$ for all x in the domain of f.
The smallest such positive number p is the period of f.

Remarks :

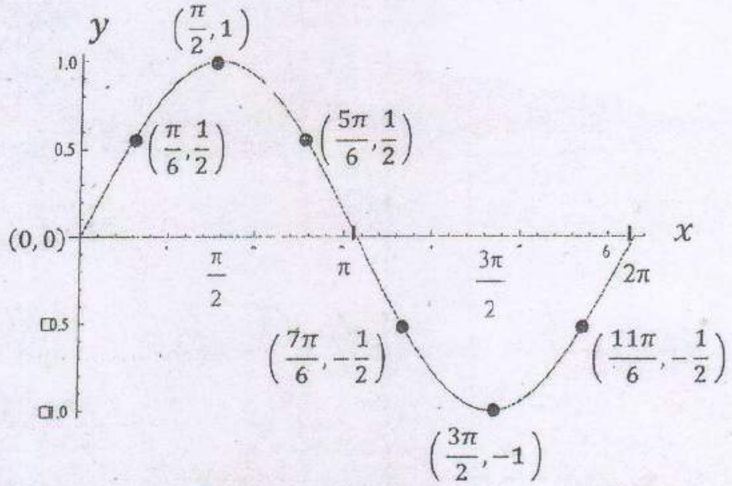
1) The functions $\sin x$, $\cos x$, $\sec x$, and $\csc x$ are periodic functions with period 2π.

2) The functions $\tan x$ and $\cot x$ are periodic functions with period π.

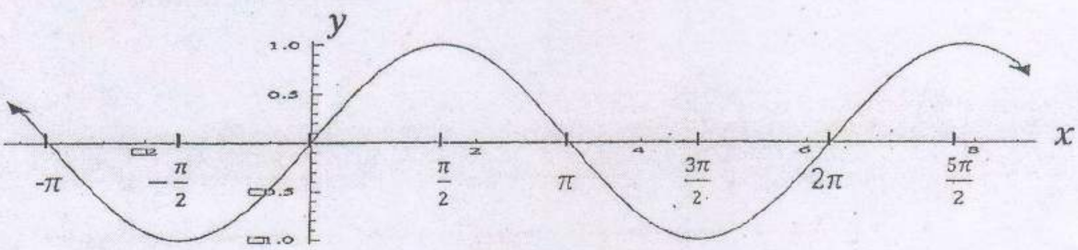
2.5.2: The Graph of $\sin x$

The graph of the function $y = \sin x$ is the line passing through all the points $(x, \sin x)$ on the xy-plane .

The graph of the function $y = \sin x$ for the interval $[0, 2\pi]$ is the line passing through the points $(0, 0)$, $(\frac{\pi}{6}, \frac{1}{2})$, $(\frac{\pi}{2}, 1)$, $(\frac{5\pi}{6}, \frac{1}{2})$, $(\pi, 0)$, $(\frac{7\pi}{6}, -\frac{1}{2})$, $(\frac{3\pi}{2}, -1)$, $(\frac{11\pi}{6}, -\frac{1}{2})$, and $(2\pi, 0)$ which is shown in the following figure



The graph of the function $y = \sin x$ is shown in the following figure



The period of the function $y = \sin x$ is 2π . The domain of the function $y = \sin x$ is the set of all real numbers R .

The range of the function $y = \sin x$ is the interval $[-1, 1]$.

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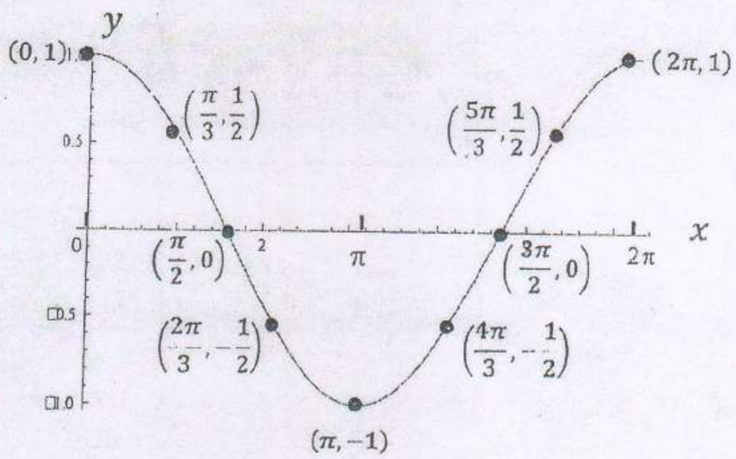
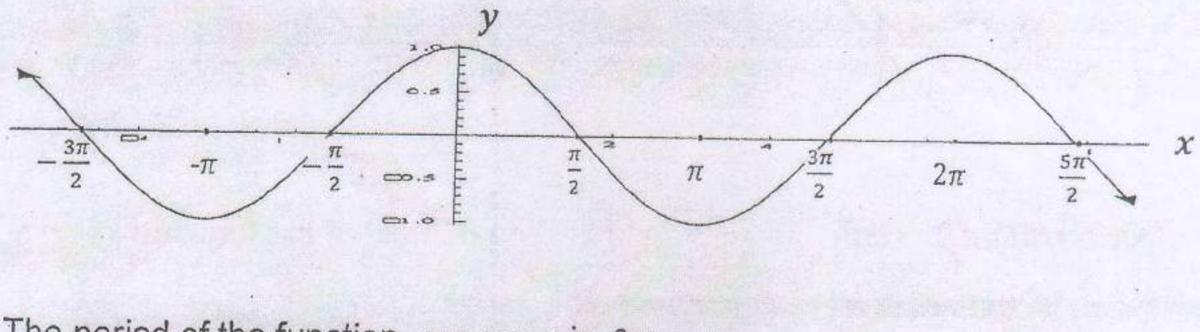
2.5.3: The Graph of $\cos x$

The graph of the function $y = \cos x$ is the line passing through all the points $(x, \cos x)$ on the xy-plane .

The graph of the function $y = \cos x$ for the interval $[0, 2\pi]$ is the line Passing through the points $(0, 1)$, $(\frac{\pi}{3}, \frac{1}{2})$, $(\frac{\pi}{2}, 0)$, $(\frac{2\pi}{3}, -\frac{1}{2})$, $(\pi, -1)$, $(\frac{4\pi}{3}, -\frac{1}{2})$, $(\frac{3\pi}{2}, 0)$, $(\frac{5\pi}{3}, \frac{1}{2})$, and $(2\pi, 1)$ which is shown in the following figure



The graph of the function $y = \cos x$ is shown in the following figure



The period of the function $y = \cos x$ is 2π .
The domain of the function $y = \cos x$ is the set of all real numbers R .
The range of the function $y = \cos x$ is the interval $[-1, 1]$.

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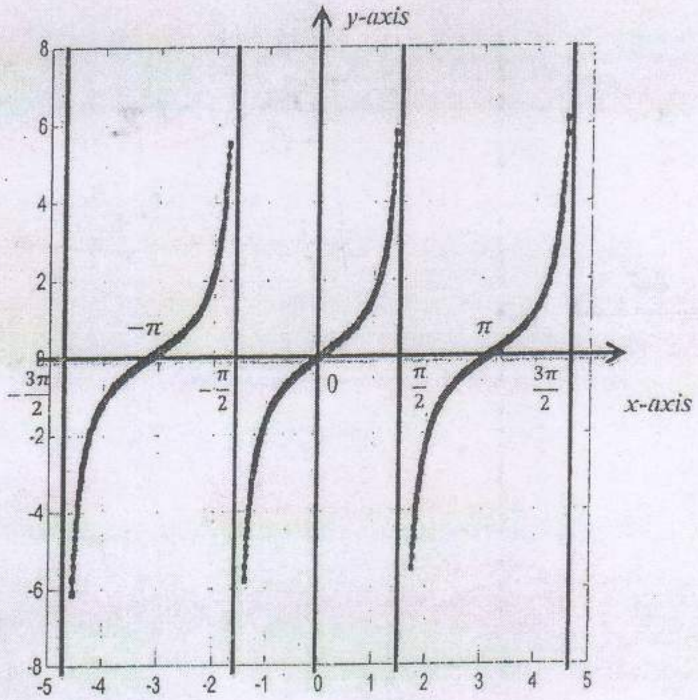
2.5.4: The Graphs of $\tan x$ and $\sec x$

The graph of the function $y = \tan(x)$ is the line passing through all

the points $(x, \tan x)$ on the xy-plane .

The graph of $y = \tan(x)$ is shown in the following figure



The graph of $y = \sec(x)$ is shown in the following figure