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## 2- Gauss-Jordan elimination:

In this method, the augmented matrix is converted as shown:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \vdots & C_1 \\ a_{21} & a_{22} & a_{23} & \vdots & C_2 \\ a_{31} & a_{32} & a_{33} & \vdots & C_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & C_1' \\ 0 & 1 & 0 & \vdots & C_2' \\ 0 & 0 & 1 & \vdots & C_3' \end{bmatrix}$$

In this case, we have:

$$1 * X_1 = C_1' \Rightarrow X_1 = C_1'$$

$$1 * X_2 = C_2' \Rightarrow X_2 = C_2'$$

$$1 * X_3 = C_3' \Rightarrow X_3 = C_3'$$

Elimination procedure:-

\* The augmented matrix:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \vdots & C_1 \\ a_{21} & a_{22} & a_{23} & \vdots & C_2 \\ a_{31} & a_{32} & a_{33} & \vdots & C_3 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

\* To eliminate  $a_{21}$  and  $a_{31}$ :

First divide  $R_1$  by  $a_{11}$ : New  $R_1 = R_1 / a_{11}$

we get

$$\begin{bmatrix} 1 & a'_{12} & a'_{13} & \vdots & C_1' \\ a_{21} & a_{22} & a_{23} & \vdots & C_2 \\ a_{31} & a_{32} & a_{33} & \vdots & C_3 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

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\* New  $R_2 = R_2 - R_1 * a_{21}$

New  $R_3 = R_3 - R_1 * a_{31}$

we get :

$$\begin{bmatrix} 1 & a'_{12} & a'_{13} & \vdots & c'_1 \\ 0 & a'_{22} & a'_{23} & \vdots & c'_2 \\ 0 & a'_{32} & a'_{33} & \vdots & c'_3 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

\* To eliminate  $a_{12}$  and  $a_{32}$

First divide  $R_2$  by  $a'_{22}$  : New  $R_2 = R_2 / a'_{22}$

we get :

$$\begin{bmatrix} 1 & a'_{12} & a'_{13} & \vdots & c'_1 \\ 0 & 1 & a''_{23} & \vdots & c''_2 \\ 0 & a'_{32} & a'_{33} & \vdots & c'_3 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

\* New  $R_1 = R_1 - R_2 * a'_{12}$

New  $R_3 = R_3 - R_2 * a'_{32}$

we get :

$$\begin{bmatrix} 1 & 0 & a''_{13} & \vdots & c''_1 \\ 0 & 1 & a''_{23} & \vdots & c''_2 \\ 0 & 0 & a''_{33} & \vdots & c''_3 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

\* To eliminate  $a''_{13}$  and  $a''_{23}$  :

New  $R_3 = R_3 / a''_{33}$  , we get

$$\begin{bmatrix} 1 & 0 & a'''_{13} & \vdots & c'''_1 \\ 0 & 1 & a'''_{23} & \vdots & c'''_2 \\ 0 & 0 & 1 & \vdots & c'''_3 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

$$* \text{New } R_1 = R_1 - R_3 * a_{13}''$$

$$\text{New } R_2 = R_2 - R_3 * a_{23}''$$

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\* we get

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & C_1''' \\ 0 & 1 & 0 & C_2''' \\ 0 & 0 & 1 & C_3''' \end{array} \right]$$

\* The solution are:  $x_1 = C_1'''$ ,  $x_2 = C_2'''$ ,  $x_3 = C_3'''$ .

Example Solve the following system of linear equations by using Gauss-Jordan elimination method?

$$3x_1 - 6x_2 + 7x_3 = 3$$

$$9x_1 - 5x_3 = 3$$

$$5x_1 - 8x_2 + 6x_3 = -4$$

Sol. The augmented matrix is:

$$\left[ \begin{array}{ccc|c} 3 & -6 & 7 & 3 \\ 9 & 0 & -5 & 3 \\ 5 & -8 & 6 & -4 \end{array} \right] \begin{array}{l} R_1 = R_1/3 \\ R_2 \\ R_3 \end{array} \Rightarrow$$

$$\left[ \begin{array}{ccc|c} 1 & -2 & 2.333 & 1 \\ 9 & 0 & -5 & 3 \\ 5 & -8 & 6 & -4 \end{array} \right] \begin{array}{l} R_1 \\ R_2 = R_2 - 9R_1 \\ R_3 = R_3 - 5R_1 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & -2 & 2.333 & 1 \\ 0 & 18 & -25.997 & 3 \\ 0 & 2 & -5.665 & -4 \end{array} \right] \begin{array}{l} R_1 \\ R_2 = R_2/18 \\ R_3 \end{array} \Rightarrow$$

$$\left[ \begin{array}{ccc|c} 1 & -2 & 2.333 & 1 \\ 0 & 1 & -1.444 & -0.333 \\ 0 & 2 & -5.665 & -9 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$R_1 = R_1 - (-2)R_2, R_3 = R_3 - 2R_2$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & -0.555 & 0.334 \\ 0 & 1 & -1.444 & -0.333 \\ 0 & 0 & -2.777 & -8.334 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 = R_3 / -2.777 \end{array}$$

$$R_3 = R_3 / -2.777$$

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$$\begin{bmatrix} 1 & 0 & -0.555 & : & 0.334 \\ 0 & 1 & -1.444 & : & -0.333 \\ 0 & 0 & 1 & : & 3.001 \end{bmatrix} \begin{array}{l} R_1 = R_1 - (-0.555)R_3 \\ R_2 = R_2 - (-1.444)R_3 \\ R_3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & : & 1.999 \\ 0 & 1 & 0 & : & 4 \\ 0 & 0 & 1 & : & 3.001 \end{bmatrix}$$

∴ The solution are

$$x_1 = 1.999, x_2 = 4, x_3 = 3.001$$

### Exercise

Solve the following system of linear equations by using Gauss-Jordan elimination method:

$$\begin{array}{l} x_1 - x_2 + x_3 = -4 \\ 5x_1 - 4x_2 + 3x_3 = -12 \\ 2x_1 + x_2 + x_3 = 11 \end{array}$$