Exercise:- (16)

Solve the following equation using false position method:

$f(x) = x^3 - 2x^2 - x - 1$ in the interval $[-1, 0]$ and $\epsilon = 0.001$?

3- Newton-Raphson method:-

Let $f(x)$ be a continuous and differentialbe function, x_0 the intial approximate value of the root to the equation. therefore a new root can be defined as follows:

$$x_1 = x_0 + h$$

where h is the correction value of the root:

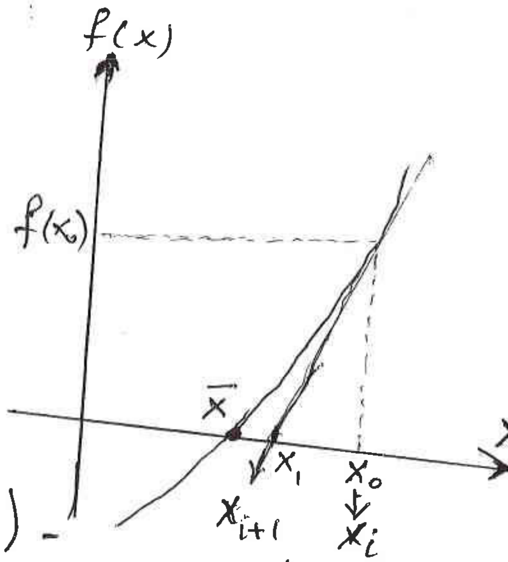
$$f(x_1) = f(x_0 + h)$$

by using Taylor's formula at the point x_0, we get:

$$f(x_1) = f(x_0) + (x_1 - x_0)\frac{f'(x_0)}{1!} + (x_1 - x_0)^2 \frac{f''(x_0)}{2!} + \cdots$$

by iteration this proceedure, we will get a general formula of the Newton-Raphson method:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, \quad i = 0, 1, 2, \cdots$$



The algorithm of the method:-

1- Take an initial value (x_0).

2. Calculate $f(x_0)$ and $f'(x_0)$

3. Calculate the intersection (x_1).

4. Put $x_0 = x_1$ and Calculate the new intersection x_2 by the same procedure.

5. Repeat the process to get $x_3, x_4, \ldots$ until reaching the required accurracy.

* in general $\quad x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)} \quad , i = 0, 1, 2, \ldots$

Example 1. :

Find the root of $f(x) = e^x - 3x$ in the interval $[0,1]$. Correct to $|x_{i+1} - x_i| < 0.005$, use Newton-Raphson method with $x_0 = 0$.

Solution:-

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x_i) = e^{x_i} - 3x_i \quad , \quad f'(x) = e^x - 3$$

$$x_{i+1} = x_i - \frac{e^{x_i} - 3x_i}{e^{x_i} - 3} \quad , i = 0, 1, 2, \ldots$$

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}	$\lvert x_{i+1}-x_i \rvert$
0	0	1	-2	0.5	—
1	0.5	0.1487	-1.3512	0.61	0.11
2	0.61	0.0104	-1.1595	0.6156	0.0056
3	0.6156	$3.97 * 10^{-3}$	-1.1492	0.619	0.0034

$\therefore$ The root $\bar{x} \simeq 0.619$.

Example 2:- Find the root of the equation: $(x-2)^2 = x + 54$ by N-R method correct to two loops, use $x_0 = 8$?

Solution

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, \quad i = 0, 1, 2, \ldots$$

$$f(x_i) = x_i^2 - 5x_i - 50$$

$$f'(x_i) = 2x_i - 5$$

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}
0	8	-26	11	10.3636
1	10.3636	5.5862	15.7272	10.0084
2	10.0084	0.1260	15.0168	10.0000

$\therefore \bar{x} = 10.0000$