The combination of various surfaces of thin lenses will determine the signs of the corresponding spherical radii.

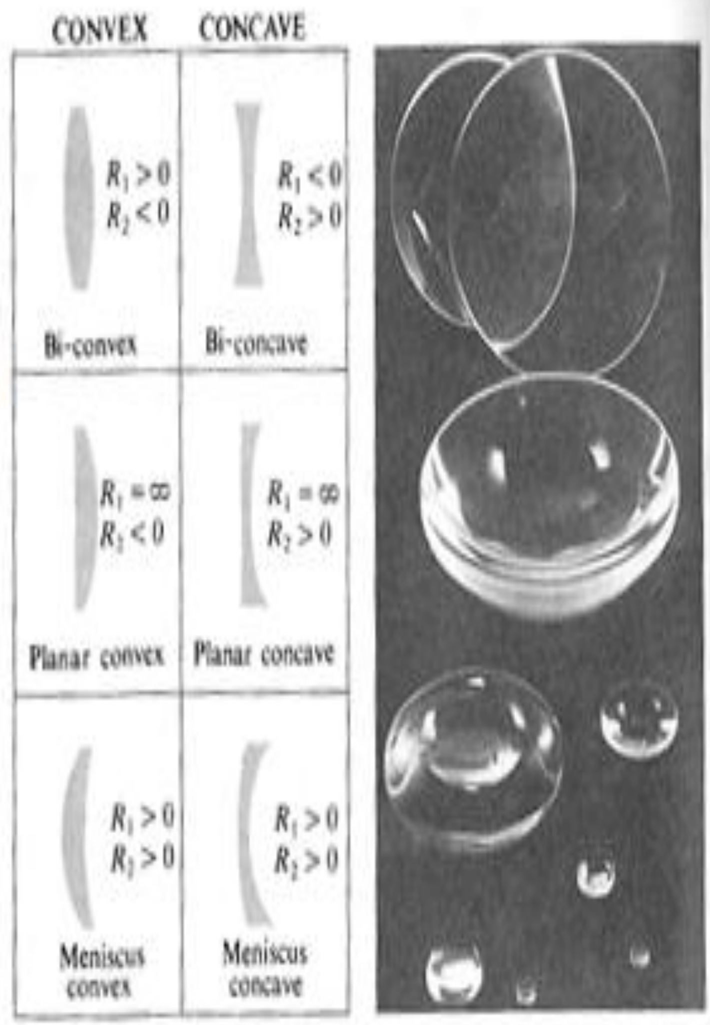
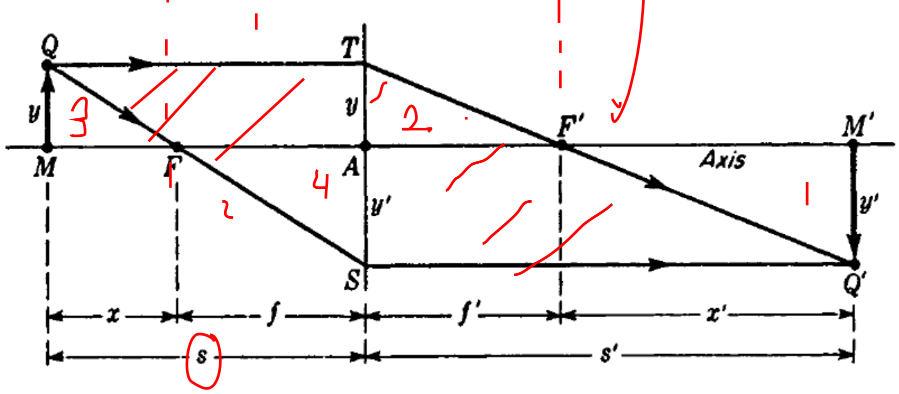


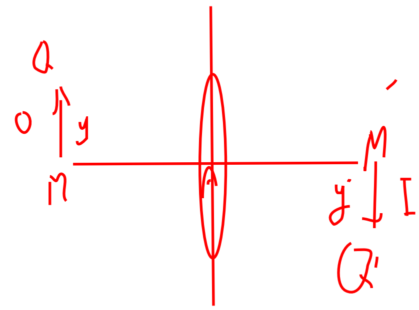
Figure (1): deferent thin lenses will determine the sign radii of curvature.

4.10 DERIVATION OF THE LENS FORMULA

A diagram for derivation the equation 4.1 (**lens formula**) is presented in figure 4.17 , which shows only two rays leading from the object of height *y* to the image of height *y'*. Let sand *s'* represent the object and image distances from the lens center and *x* and *x'* their respective distances from the focal points *F* and *F'*. From similar triangles *Q'TS* and *F' TA* the proportionality between corresponding sides gives

$$\frac{y - y'}{s'} = \frac{y}{f'} \qquad (4.11)$$





Figure(4.17): The geometry used for the derivation of thin-lens formulas.

$\Delta QTS \cong FAS \Rightarrow$

$$\frac{y - y'}{s} = \frac{-y'}{f} \qquad (4.12)$$

Note that *y - y'* is written instead of *y + y'* because *y'*, by the convention of signs,is a negative quantity. From the similar triangles *QTS* and *FAS*, The (4.11 and 4.12) sum of these two equations is

$$\frac{y \cancel{-} y'}{s} + \frac{y \cancel{-} y'}{s'} = \frac{\cancel{y}}{f'} - \frac{y'}{f} \qquad (4.13)$$

Since $f = f'$,the two terms on the right can be combined and *y - y'* canceled out,yielding the desired equation,

$f = f'$

$$\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}} \qquad (4.14)$$

This equation is called Gaussian form or the lens equation of thin lenses, can be used to relate the image distance and object distance for a thin lens.

Another form of the lens formula is the Newtonian form, is obtained in an analogous way from two other sets of similar triangles, *QMF* and *FAS* on the one hand and *TAF'* and *F'M'Q'* on the other. We find 3

4

$$\frac{y}{x} = \frac{-y'}{f} \quad \text{and} \quad \frac{-y'}{x'} = \frac{y}{f} \qquad (4.15)$$

$\rightarrow f = f'$

Multiplication of one equation by the other gives

$$xx' = f^2$$

In the Gaussian formula the object distances are measured from the center of lens, while in the Newtonian formula they are measured from the focal points. Object distances (s or x) are positive if the object lies to the left of its reference point (A or F, respectively), while image distances (s' or x') are positive if the image lies to the right of its reference point (A or F', respectively).

The lateral magnification as given by Eq. (4c) corresponds to the Gaussian form.

When distances are measured from focal points, one should use the Newtonian form, which can be obtained directly from equation (4.10)

$$m = \frac{y'}{y} = -\frac{f}{x} = -\frac{x'}{f} \qquad (4.16)$$

In the more general case where the medium on the two sides of the lens is different, it will be shown in the next section that the primary and secondary focal distances f and f' are different, being in the same ratio as the two refractive indices. The Newtonian lens formula then takes the symmetrical form

$$xx' = ff'$$

The result is that the object and image must be on the opposite sides of their respective focal points.

Newtonian Form:

$x > 0$ if the object is to the **left** of F.

$x' > 0$ if the image is to the **right** of F'.

The result is that the object and image must be on the opposite sides of their respective focal points.

$M_T > 0 \Rightarrow$ Erect image and $M_T < 0 \Rightarrow$ Inverted image. All real images for a thin lens will be inverted

The power

A lens is tools produce an image by refraction of light at two boundary surfaces. Lens is made of plastic ,glasses, quartz, fused and silica.

The power of a the lens is measure of the ability to produce convergence rays or parallel rays .A convex lenses is distinguished of small focal length and high power produces a large convergence effect . a convex lens is a positive value of the power.

The concave lens is negative power produces divergence,.

F (focal length measured in meters, . power P are in units called diopters

$$\text{Power p(diopeter)} = \frac{1}{f(meter)}$$

Lens Marker's Equation

If a lens is to be ground to some specified focal length, the refractive index of the glass must be known. Supposing the index to be known, the radii of curvature must be so chosen as to satisfy the equation

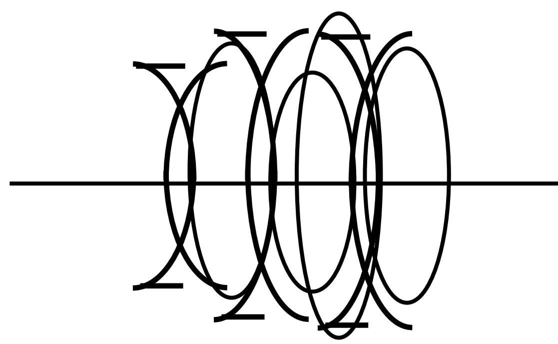
$$\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

If a lens is to be ground to some specified focal length, the refractive index of the glass must be known. Supposing the index to be known, the radii of curvature must be so chosen as to satisfy the equation

$$\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Suppose that we have in general a system of N lenses whose thicknesses are small and each lens is placed in contact with its neighbor

$$\frac{1}{f_{ef}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \cdots \frac{1}{f_N}$$



a) If the optical system is consist of two lenses is contact

The effective focal length of combination of two lenses f1 and f2 is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

The power of two lensesP1 and P2 or more ,placed in contact is the sum of two lenses

P=P1+ P2

Example (3): A meniscus(concave-convex)lens has an index of refraction 1.5 and the radii of curvature of its surface are 10 and 20 cm .the concave surface is upward and it is filled with an oil of refractive index 1.6 .calculate the focal length of oil-glass combination

Solution:

For the glass

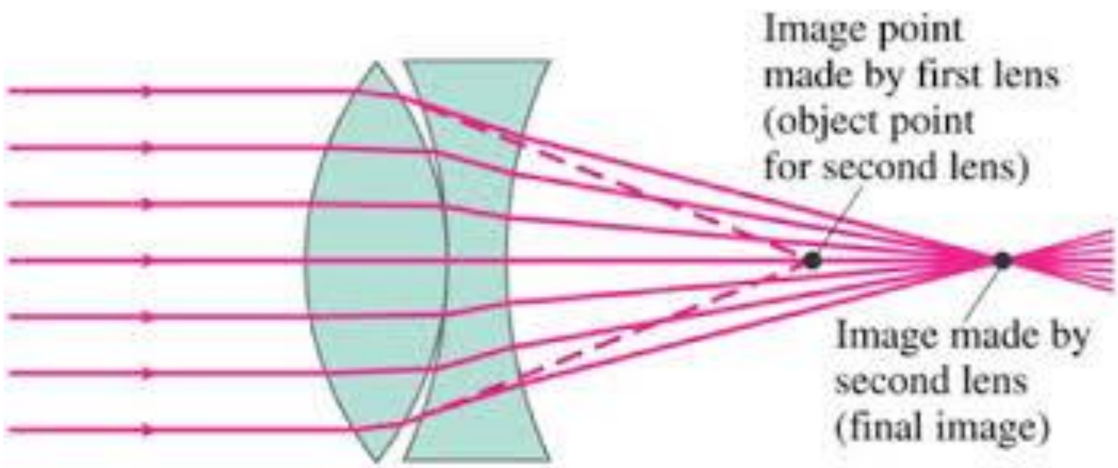
$$\frac{1}{f_1} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\frac{1}{f_1} = (1.5-1)\left(\frac{1}{10\ cm} - \frac{1}{20\ cm}\right) = \frac{1}{40\ cm}$$

$$\frac{1}{f_2} = (1.6-1)\left(\frac{1}{20\ cm} - \overset{0}{\cancel{\frac{1}{\infty}}}\right) = \frac{3}{100\ cm}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{40\ cm} + \frac{3}{100\ cm} = \frac{11}{200\ cm}$$

$$\therefore f = 18.2\ \overset{cm}{\cancel{m}}$$



2) Two lens combination separated by distance d

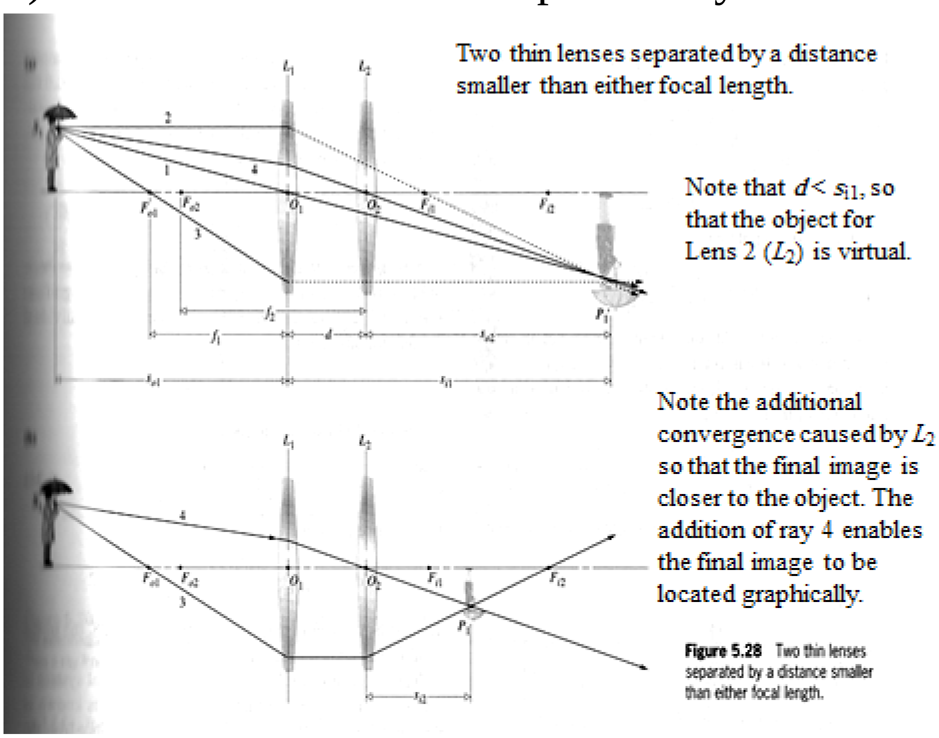


Figure 5.28 Two thin lenses separated by a distance smaller than either focal length.

a) Note that $d < s_{i1}$, so that the object for Lens 2 (L_2) is virtual

Solution:

Step 1 :

$$\frac{1}{s_1} + \frac{1}{s'_1} = \frac{1}{f_1}$$

$$s'_{i_1} = \frac{f_1 x s_1}{s_1 - f_1} \Rightarrow \text{Image}$$

2) step 2 $\quad \rightarrow s_{o2} = d - s'_{i1}$ 2

3) $s''_{i_2} = \dfrac{f_2 x \; s_{o2}}{s_{o2} - f_2}$ *the final image*

4) $m_{t=} m_1 m_2$ the size of final image

5) The focal length effective equation of two lenses separated at distance d or apart between them

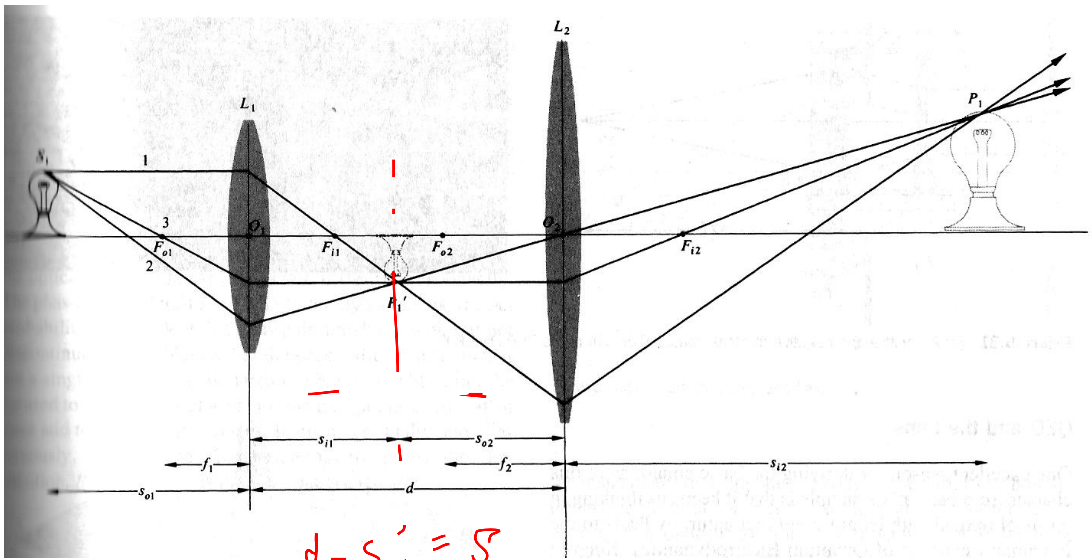
$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$

6) Power

When two lenses of focal lenses f_1 and f_2 are placed coaxial and separated by a distance d the combined power

$$P = P_1 + P_2 - d\, P_1 P_2$$

3. Note that $d > s_{i1}$, so that the object for Lens 2 (L_2) is real



$d - s_{i1} = s_{o2}$