

6+Chapter five (Mirror Formula), Power of Mirrors, Thick mirror, Thick – Mirror Formulas,

Other thick Mirrors, problems.

Chapter 5: SPHERICAL MIRRORS:

A spherical reflecting surface has image-forming properties similar to those of a thin lens or of a single refracting surface.

Spherical mirrors are divided into two types:

- 1- Convex mirror
- 2- Concave mirror

FOCAL POINT AND FOCAL LENGTH

Diagrams showing the reflection of a parallel beam of light by a concave mirror and by a convex one are given in Fig. 6A.

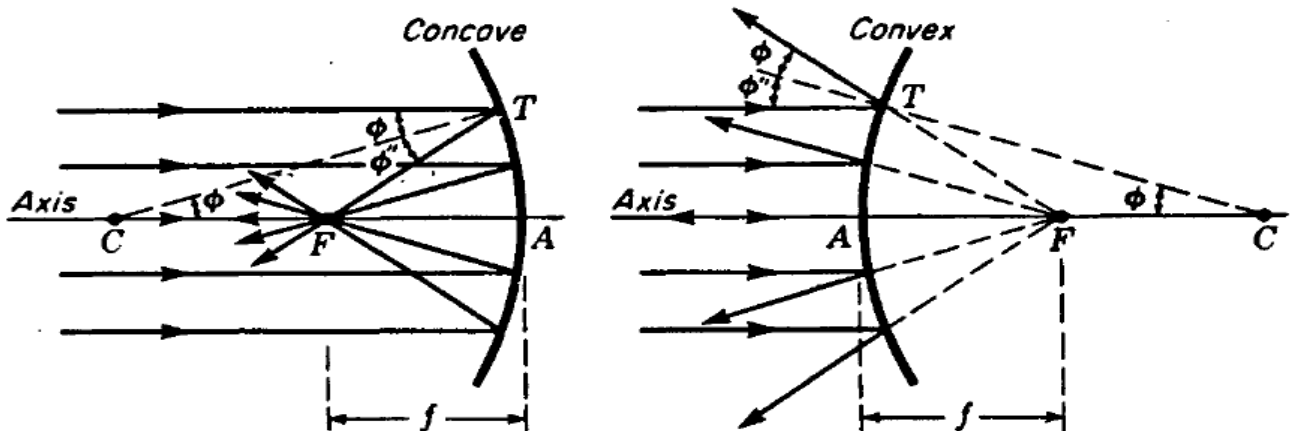


FIGURE 6A

The primary and secondary focal points of spherical mirrors coincide.

A ray striking the mirror at some point such as T obeys the law of reflection $\phi'' = \phi$. All rays are shown as brought to a common focus at F , although this will be strictly true only for paraxial rays. The point F is called the *focal point* and the distance FA the *focal length*. In the second diagram the reflected rays diverge as though they came from a common point F . Since the angle TCA also equals ϕ , the triangle TCF is isosceles, and in general $CF = FT$. But for very small angles ϕ (paraxial rays), FT approaches equality with FA . Hence $FA = 1/2(CA)$

$$f = -1/2 r \quad (6a)$$

and the focal length equals one-half the radius of curvature [see also Eq. (6d)].

The Convention of Signs:

- 1- The drawing is from left to right
- 2- The focal length is positive (+) for concave mirror and negative (-) in convex mirror.
- 3- The radius of curvature is positive (+) for concave mirror and negative (-) in convex mirror.
- 4- The object and image distance are positive in the left and negative in the right.

GRAPHICAL CONSTRUCTIONS

The parallel-ray method of construction is given for a concave mirror in Fig. 6E. Three rays leaving Q are, after reflection, brought to the conjugate point Q' .

The image is real, inverted, and smaller than the object. Ray 4 drawn parallel to the axis is, by definition of the focal point, reflected through F . Ray 6 drawn through F is reflected parallel to the axis, and ray 8 through the center of curvature strikes the mirror normally and is reflected back on itself. The crossing point of any two of these rays is sufficient to locate the image.

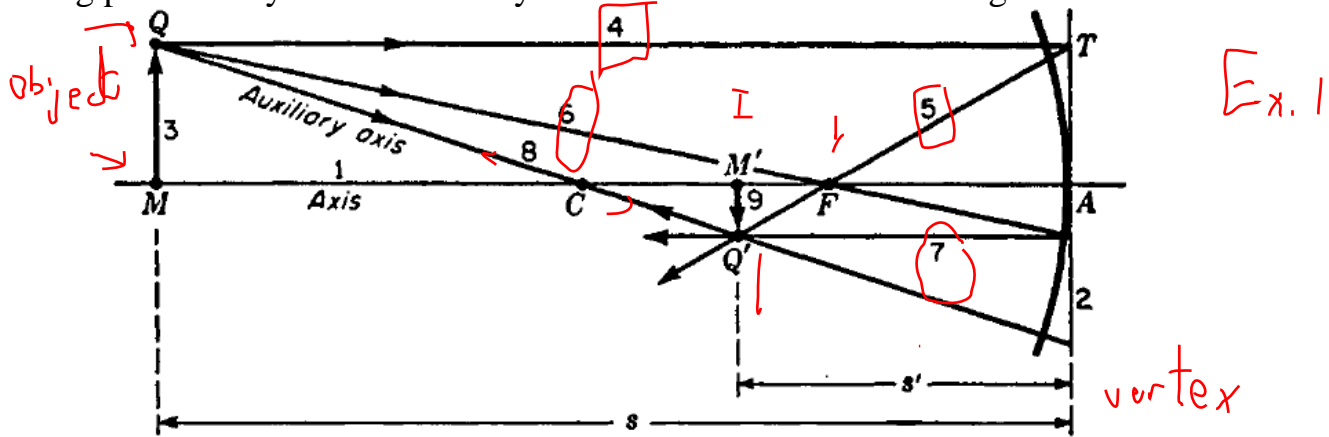


FIGURE 6E

Parallel-ray method for graphically locating the image formed by a concave mirror.

A similar procedure is applied to a convex mirror in Fig. 6F. The rays from the object point Q , after reflection, diverge from the conjugate point Q' . Ray

4, starting parallel to the axis, is reflected as if it came from F . Ray 6 toward the center of curvature C is reflected back on itself, while ray 7 going toward F is reflected parallel to the axis. Since the rays never pass through Q' , the image $Q'M'$ in this case is virtual.

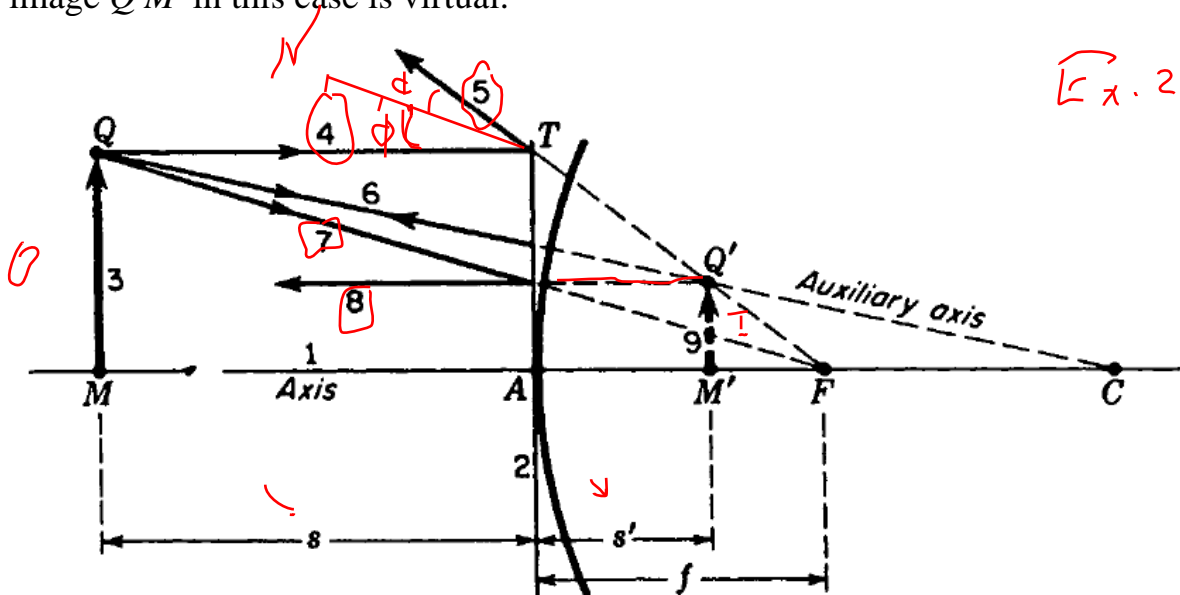


FIGURE 6F

Parallel-ray method for graphically locating the image formed by a convex mirror.

were parallel incident rays, they would come to a focus at P in the focal plane. If in place of ray 4 another ray were drawn through C and parallel to ray 3, it too would cross the focal plane at P . A ray through the center of curvature would be reflected directly back upon itself.

MIRROR FORMULAS

In order to be able to apply the standard lens formulas of the preceding chapters to spherical mirrors with as little change as possible, we must adhere to the following sign conventions:

- 1 Distances measured from left to right are positive while those measured from right to left are negative.
- 2 Incident rays travel from left to right and reflected rays from right to left.
- 3 The focal length is measured from the focal point to the vertex. This gives f a positive sign for concave mirrors and a negative sign for convex mirrors.
- 4 The radius is measured from the vertex to the center of curvature. This makes r negative for concave mirrors and positive for convex mirrors.
- 5 Object distances s and image distances s' are measured from the object and from the image respectively to the vertex. This makes both s and s' positive and the object and image real when they lie to the left of the vertex; they are negative and virtual when they lie to the right.

The last of these sign conventions implies that for mirrors the object space and the image space coincide completely, the actual rays of light always lying in the space to the left of the mirror. Since the refractive index of the image space is the same as that of the object space, the n' of the previous equations becomes numerically equal to n .

The following is a simple derivation of the formula giving the conjugate relations for a mirror. In Fig. 6G it is observed that by the law of reflection the radius CT bisects the angle MTM' . Using a well-known geometrical theorem, we can then write the proportion $\frac{MC}{MT} = \frac{CM'}{M'T}$

Now, for paraxial rays, $MT \sim MA = s$ and $M'T \sim M'A = s'$, where the symbol \sim means "is approximately equal to." Also, from the diagram,

$$MC = MA - CA = s + r$$

$$\text{and } CM' = CA - M'A = -r - s' = -(s' + r)$$

Substituting in the above proportion gives $\frac{s+r}{s} = -\frac{s'+r}{s'}$ which can easily be put in the form:

$$\frac{1}{s} + \frac{1}{s'} = -\frac{2}{r} \quad (6b) \text{ Mirror formula}$$

The primary focal point is defined as that axial object point for which the image is formed at infinity, so substituting $s = f$ and $s' = \infty$ in Eq. (6b), we have

$$\frac{1}{f} + \frac{1}{\infty} = -\frac{2}{r}$$

$$\text{from which } \frac{1}{f} = -\frac{2}{r} \text{ or } f = -\frac{r}{2} \quad (6e)$$

The secondary focal point is defined as the image point of an infinitely distant object point. This is $s' = f'$ and $s = \infty$, so that

$$\frac{1}{\infty} + \frac{1}{f'} = -\frac{2}{r}$$

$$\text{from which } \frac{1}{f'} = -\frac{2}{r} \text{ or } f' = -\frac{r}{2} \quad (6d)$$

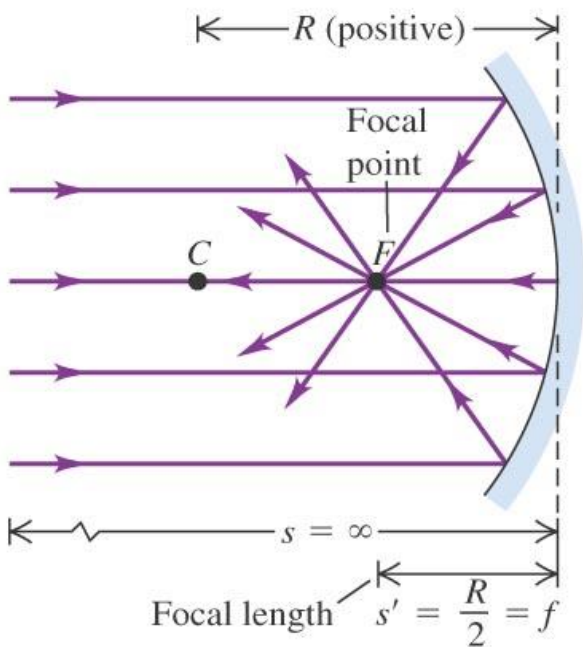
Therefore the primary and secondary focal points fall together, and the magnitude of the focal length is one-half the radius of curvature. When $-2/r$ is replaced by $1/f$, Eq. (6b) becomes

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad (6c) \text{ just as for lenses.}$$

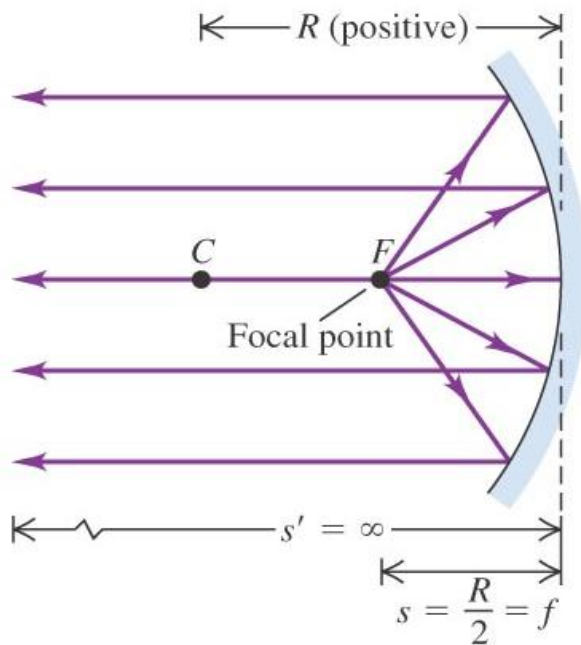
The lateral magnification of the image from a mirror can be evaluated from the geometry of Fig. 6C. From the proportionality of sides in the similar triangles $Q'AM'$ and QAM we find that $\frac{y'}{y} = -\frac{s'}{s}$, giving

$$m = \frac{y'}{y} = -\frac{s'}{s} \quad (6f)$$

(a) All parallel rays incident on a spherical mirror reflect through the focal point.



(b) Rays diverging from the focal point reflect to form parallel outgoing rays.



EXAMPLE 1 An object 2.0 cm high is situated 10.0 cm in front of a concave mirror of radius 16.0 cm. Find (a) the focal length of the mirror, (b) the position of the image, and (c) the lateral magnification?

SOLUTION

The given quantities are $y = +2.0$ cm, $s = +10.0$ cm, and $r = -16$ cm. The unknown quantities are f , s' , and m . (a) By Eq. (6c),

$$f = -\frac{-16}{2} = +8 \text{ cm}$$

(b) By Eq. (6e), $\frac{1}{10} + \frac{1}{s'} = \frac{1}{8}$ or $\frac{1}{s'} = \frac{1}{8} - \frac{1}{10} = \frac{1}{40}$ \Leftarrow

Giving $s' = +40$ cm

(c) By Eq. (6f), $m = -\frac{40}{10} = -4$ \Leftarrow $m = -\frac{5}{1}$

$$\frac{1}{5} + \frac{1}{s'} = \frac{1}{f}$$

The image occurs 40.0 cm to the left of the mirror, is 4 times the size of the object, and is real and inverted.

POWER OF MIRRORS

The power notation that was used in Sec. 4.12 to describe the image-forming properties of lenses can readily be extended to spherical mirrors as follows.

As definitions

let

$$P = \frac{1}{f(m)}$$

$$M = \frac{y'}{y}$$

$$r = +ve$$

EXAMPLE 2 an object is located 20.0 cm in front of a convex mirror of radius 50.0 cm. Calculate (a) the power of the mirror, (b) the position of the image, and (c) its magnification.

SOLUTION Expressing all distances in meters, we have

$$P = \frac{1}{f(m)} = 4 \text{ D}$$

$$\Rightarrow f = -r/2 = -25 \text{ cm}$$

$$f = r/2 = 50/2 = 25 \text{ cm}$$

$$S' = \frac{sx}{s-x} = \frac{-25 \times 20}{-25 + 20} = -11.11 \text{ cm}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$c) m = -s'/s = 11.11/20 = +0.555$$

$$f = -ve$$

negative

$$m = +ve$$

erect

min

THICK MIRRORS

The term *thick mirror* is applied to a lens system in which one of the spherical surfaces is a reflector. Under these circumstances the light passing through the system is reflected by the mirror back through the lens system, from which it emerges finally into the space from which it entered the lens.

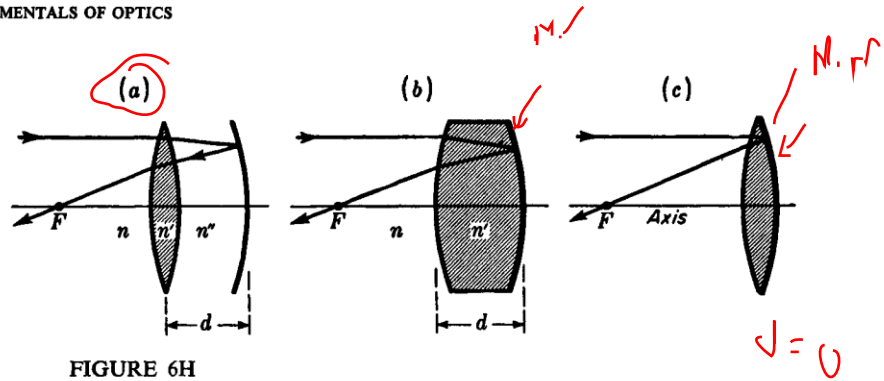


FIGURE 6H
Diagrams of several types of thick mirrors, showing the location of their respective focal points.

In the first illustration the lens is considered *thin* so that its own principal points may be assumed to coincide at H_1 , its center. An incident ray parallel to the axis is refracted by the lens, reflected by the mirror, and again refracted by the lens before it crosses the axis of the system at F . The point T where the incident and final rays, when extended, cross each other locates the principal plane, and H represents the principal point. If we follow the sign conventions for a single mirror (Sec. 6.3), the focal length f of this particular combination is positive and is given by the interval FH .

In the second illustration (Fig. 6J) the incident ray is refracted by the first surface, reflected by the second, and finally refracted a second time by the first surface to a point F where it crosses the axis. The point T where the incident and final rays intersect locates the principal plane and principal point H .

The graphical ray-tracing construction for this case, shown in the auxiliary diagram in Fig. 6J, is started by drawing XZ parallel to the axis. With the origin O near the center, intervals proportional to n and n' are measured off in both directions

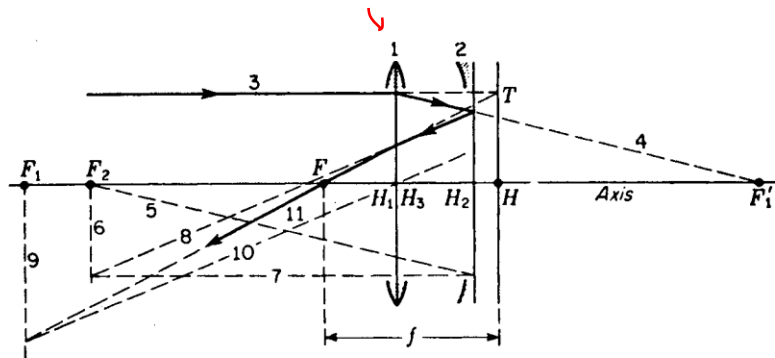


FIGURE 6J
Oblique-ray construction for locating the focal point and principal point of a thick mirror.

6.6 THICK-MIRROR FORMULAS

These formulas will be given in the power notation for case (a) shown in Fig. 6H. When r_1 , r_2 , and r_3 are the radii of the three surfaces consecutively from left to right, the power of the combination can be shown* to be given by

$$\bullet \quad P = (1 - cP_1)(2P_1 + P_2 - cP_1P_2) \quad (6l)$$

where, for the case in diagram (a) only and $n'' = n$,

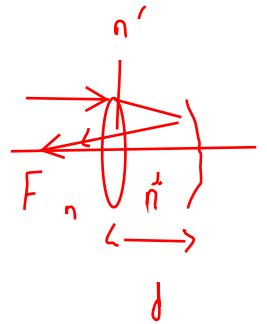
$$P_1 = (n' - n)(K_1 - K_2) \quad (6m)$$

$$P_2 = -2nK_3 \quad (6n)$$

and

$$K_1 = \frac{1}{r_1} \quad K_2 = \frac{1}{r_2} \quad K_3 = \frac{1}{r_3}$$

* For a derivation of these equations, see J. P. C. Southall, "Mirrors, Prisms, and Lenses," 3d ed., p. 379, The Macmillan Company, New York, 1936.



eq 1, 2

[see Eqs. (4p) and (6d)]. Of the refractive indices, n' represents that of the lens and n that of the surrounding space. The distance from the lens to the principal point of the combination is given by

$$H_1H = \frac{c}{1 - cP_1} \quad (6o)$$

$$c = \frac{d}{n}$$

It is important to note from Eq. (6o) that the position of H is independent of the power P_2 of the mirror and therefore of its curvature K_3 .

EXAMPLE 3 A thick mirror like that shown in Fig. 6H(0) has as one component a thin lens of index $n' = 1.50$, radii $r_1 = +50.0$ cm, and $r_2 = -50.0$ cm. This lens is situated 10.0cm in front of a mirror of radius - 50.0 cm. Assuming that air surrounds both components, find (a) the power of the combination, (b) the focal length, and (c) the principal point.

SOLUTION (a) By Eq. (6m), the power of the lens is

SOLUTION (a) By Eq. (6m), the power of the lens is

$$P_1 = (1.50 - 1) \left(\frac{1}{0.50} - \frac{1}{-0.50} \right) = +2 \text{ D} \quad \leftarrow$$

Equation (6n) gives for the power of the mirror

$$P_2 = -2 \frac{1}{-0.50} = +4 \text{ D} \quad \leftarrow$$

From Eq. (6p),

$$\rightarrow c = \frac{d}{n} = \frac{0.10}{1} = 0.10 \text{ m}$$

Finally the power of the combination is given by Eq. (6l) as

$$\begin{aligned} P &= (1 - 0.10 \times 2)(2 \times 2 + 4 - 0.10 \times 2 \times 4) \\ &= 0.8(4 + 4 - 0.8) = +5.76 \text{ D} \end{aligned}$$

(b) A power of +5.76 D corresponds to a focal length

$$f = \frac{1}{P} = \frac{1}{5.76} = 0.173 \text{ m} = +17.3 \text{ cm}$$

(c) The position of the principal point H is determined from Eq. (6o) through the distance

$$H_1H = \frac{0.10}{1 - 0.10 \times 2} = \frac{0.10}{0.80} = 0.125 \text{ m} = +12.5 \text{ cm}$$

It is therefore 12.5 cm to the right of the lens, or 2.5 cm in back of the mirror.

OTHER THICK MIRRORS

As a second illustration of a thick mirror, consider the thick lens silvered on the back, as shown in Fig. 6H(b). A comparison of this system with the one in diagram (a) shows that Eqs. (6l) to (6p) will apply if the powers P_1 and P_2 are properly defined.

For diagram (b), P_1 refers to the power of the first surface alone, and P_2 refers to the power of the second surface as a mirror of radius r_2 in a medium of index n' . In other words,

$$\rightarrow P_1 = \frac{n' - n}{r_1} \quad P_2 = -\frac{2n'}{r_2} \quad \text{and} \quad c = \frac{d}{n'} \quad (6q) \quad \leftarrow \text{fig. (b)}$$

With these definitions the power of thick mirror (b) is given by Eq. (6l) and the principal point by Eq. (6o).

The third illustration of a thick mirror consists of a thin lens silvered on the back surface as shown in Fig. 6H(c). This system may be looked upon (1) as a special case of diagram (a), where the mirror has the same radius as the

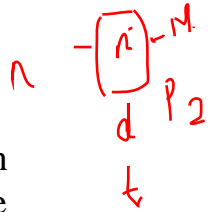
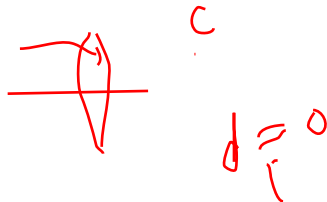


fig. (b)

back surface of the thin lens and the spacing d is reduced to zero, or (2) as a special case of diagram (b), where the thickness is reduced to practically zero. In either case Eq. (61) reduces to

$$P = (P_1 + P_2) \quad (6r)$$


The diagram shows a thin lens with a horizontal optical axis. A vertical line represents the lens. A point 'C' is marked above the lens, representing the center of curvature. A horizontal line segment labeled 'd' is drawn below the lens, with a note 'd=0' next to it, indicating the thickness of the lens is zero.

and the principal point H coincides with H' at the common center of the lens and mirror. P_1 represents the power of the thin lens in air and P_2 the power of the mirror in air, or P_1 represents the power of the first surface of radius r_1 and P_2 represents the power of the second surface as a mirror of radius r_2 in a medium of index n' [see Eq. (6q)].

SPHERICAL ABERRATION

The discussion of a single spherical mirror in the preceding sections has been confined to paraxial rays. Within this rather narrow limitation, sharp images of objects at any distance may be formed on a screen, since bundles of parallel rays close to the axis and making only small angles with it are brought to a sharp focus in the focal plane. If, however, the light is not confined to the paraxial region, all rays from one object point do not come to a focus at a common point and we have an undesirable effect known as *spherical aberration*. The phenomenon is illustrated in Fig. 6K, where parallel incident rays at increasing distances h cross the axis closer to the mirror. The envelope of all rays forms what is known as a *caustic surface*. If a small screen is placed at the paraxial focal plane F and then moved toward the mirror, a point is reached where the size of the circular image spot is a minimum. This disk-like spot is indicated in the diagram and is called the *circle of least confusion*.

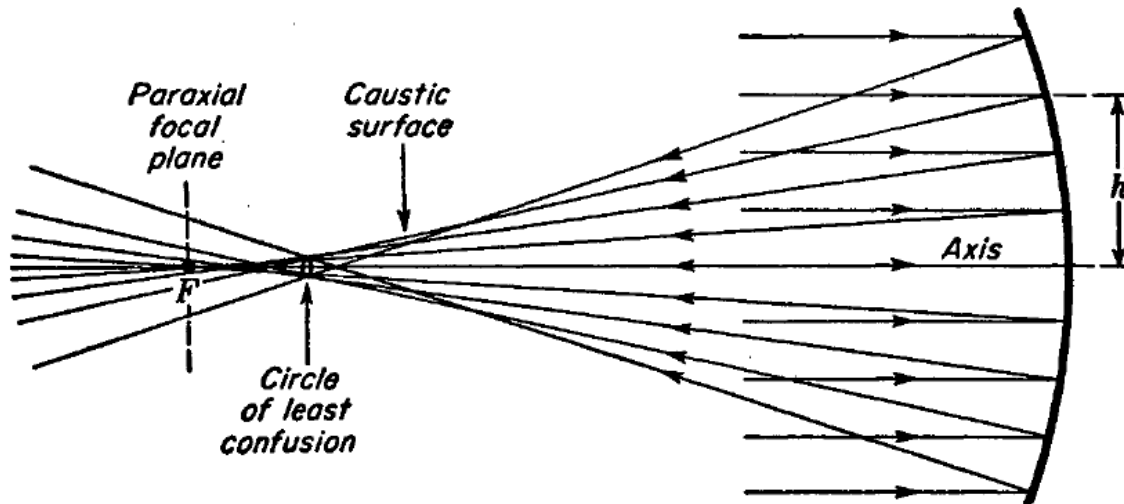


FIGURE 6K
Spherical aberration of a concave spherical mirror.

6.1 A spherical mirror has a radius of - 24.0 cm; An object 3.0 cm high is located in front of the mirror at a distance of (a) 48.0 cm, (b) 36.0 cm, (c) 24.0 cm, (d) 12.0 cm, and (e) 6.0 cm. Find the image distance for each of these object distances.

Ans. (a) + 16.0 cm, (b) + 18.0 cm, (c) + 24.0 cm, (d) + 12.0 cm, (e) - 12.0 cm

6.3 A spherical mirror has a radius of - 15.0 cm. An object 2.50 cm high is located in front of the mirror at a distance of (a) 45.0 cm, (b) 30.0 cm, (c) 15.0 cm, (d) 10.0 cm, and (e) 5.0 cm. Find the image distance for each of these object distances.

6.5 The radius of a spherical mirror is + 18.0 cm. An object 4.0 cm high is located in front of the mirror at a distance of (a) 36.0 cm, (b) 24.0 cm, and (c) 12. cm. Find the image distance and image size for each of these object distances.

Ans. (a) -7.20 cm from vertex and +0.80 cm high, (b) -6.55 cm from vertex and + 1.092 cm high, (c) - 5.40 cm from vertex and + 1.712 cm high

6.7 The radius of a spherical mirror is + 8.0 cm. An object 3.50 cm high is located in front of the mirror at a distance of (a) 16.0 cm, (b) 8.0 cm, (c) 4.0 cm, and (d) 2.0 cm.

Find the image distance and image size for each of these object distances.

6.8 Solve Prob. 6.7 graphically.

6.9 A concave mirror is to be used to focus the image of a tree on a photographic film 8.50 m away from the tree. If a lateral

magnification of $-\frac{1}{20}$ is desired, what should be the radius of curvature of the mirror? , *Ans.* - 85.2 cm