

③

$$P_5(8) = 7 + (1.6)(4) - \frac{1}{2}(1.6)(1.6-1) + \frac{2}{6}(1.6)(0.6)(-0.4) - \frac{1}{24}(1.6)(0.6)(-0.4)(-1.4) + \frac{0}{120}(1.6)(0.6)(-0.4)(-1.4)(2.4)$$

$P_5(8) = 12.769$

∴ the interpolated point (x, y) is :

$x = 8, y = 12.769$

2. Newton Backward Difference :-

* تستخدم هذه الطريقة عندما تقع قيمة (x) المطلوب استكمالها في الطرف الثاني في جدول البيانات .
 * تكتب متعددة الحدود كما يلي :-

$$P_n(x) = y_n + \frac{\nabla y_n}{1!} k + \frac{\nabla^2 y_n}{2!} k(k+1) + \frac{\nabla^3 y_n}{3!} k(k+1)(k+2) + \dots$$

where $k = \frac{x - x_n}{h}$, ∇ : Backward Difference operator

* Difference table

x_i	y_i	∇y_i	$\nabla^2 y_i$	$\nabla^3 y_i$	$\nabla^4 y_i$
x_0	y_0	∇y_1			
x_1	y_1	$y_1 - y_0$	$\nabla^2 y_2$		
x_2	y_2	∇y_2	$\nabla^2 y_3$	$\nabla^3 y_3$	
x_3	y_3	$y_2 - y_1$	$\nabla^2 y_4$	$\nabla^3 y_4$	
x_4	y_4	$y_3 - y_2$	$\nabla^2 y_4$	$\nabla^3 y_4$	
		∇y_4	$\nabla^2 y_4$	$\nabla^3 y_4$	$\nabla^4 y_0$
		$y_4 - y_3$	$\nabla y_4 - \nabla y_3$	$\nabla^2 y_4 - \nabla^2 y_3$	$\nabla^3 y_4 - \nabla^3 y_3$

④

Example: Find the value of (y) at $x=19$ for the data given below:

x_i	0	5	10	15	20	25
y_i	7	11	14	18	24	32

Solution: $n=5, h=5, k = \frac{x-25}{5}$
 for $x=19 \Rightarrow k = \frac{19-25}{5} = -1.2$

$$P_5(x) = y_5 + \frac{\nabla y_5}{1!} k + \frac{\nabla^2 y_5}{2!} k(k+1) + \frac{\nabla^3 y_5}{3!} k(k+1)(k+2) + \frac{\nabla^4 y_5}{4!} k(k+1)(k+2)(k+3) + \frac{\nabla^5 y_5}{5!} k(k+1)(k+2)(k+3)(k+4)$$

Difference table:

x_i	y_i	∇y_i	$\nabla^2 y_i$	$\nabla^3 y_i$	$\nabla^4 y_i$	$\nabla^5 y_i$
0	7					
5	11	4				
10	14	3	-1			
15	18	4	1	2		
20	24	6	2	1	-1	
25	32	8	2	0	-1	0

$\nabla y_5 = 8$
 $\nabla^2 y_5 = 2$
 $\nabla^3 y_5 = 0$
 $\nabla^4 y_5 = -1$
 $\nabla^5 y_5 = 0$

$$\therefore P_5(19) = 32 + 8(-1.2) + (-1.2)(-0.2) + \frac{(-1)}{24} (-1.2)(-0.2)$$

$$\therefore P_5(19) = 22.625$$

\therefore The interpolated point (x, y) is:

$$x = 19, y = 22.625$$

⑤

Exercises :-

① Use NDDIP to find y at $x = 2.3$ from the following data:-

x	2	4	6	8	10
y	2	1	3	8	20

② Find the value of (y) at $x = 2.5$ for the following data:-

x	0	1	2	3
y	0	1	8	135