المحاضرة السادسة : (مهم جدا)

التطبيقات الفيزيائية للاعداد والدوال العقدية .

Physical Applications

1. Wave Equation

The mathematical representation of wave is given by $y = Ae^{i(kx-wt)}$, Find the intensity I .

Solution:

$$\boxed{z = re^{i\theta}} \Longrightarrow \begin{array}{c} z \equiv y \\ r \equiv A = \text{amplitude} \\ \theta \equiv angle\ (\ or\ phase.) \equiv (kx - wt) \\ In\ PHYSICS \\ \theta\ \ is\ called\ (\boldsymbol{PHASE}\ ) \end{array}$$

$since,\ \boxed{zz^* = |z|^2 = r^2}\ \ in\ \boldsymbol{mathematics}$

$\boxed{yy^* \equiv y\bar{y} = |y|^2 = A^2}\ \ in\ \boldsymbol{physics}$

مهم

i.e. $yy^* = |y|^2 = Ae^{i(kx-wt)} * Ae^{-i(kx-wt)}$

Since ,

$$Ae^{i(kx-\omega t)} * Ae^{-i(kx-wt)} = \boldsymbol{A^2 e^{i(kx-\omega t)-i(kx-\omega t)}} = \boldsymbol{A^2 e^0}$$

$$|y|^2 = \boldsymbol{A^2} * (\boldsymbol{1}) = \boldsymbol{A^2}$$

$$\Longrightarrow |y|^2 \equiv \boldsymbol{I = A^2}$$

Activity : Find $y_1 + y_2, y_1 - y_2, y_1 y_2, |y_1 - y_2|$ for wave of the same phase of different amplitude.

Answer :

$\boldsymbol{y_1 + y_2}$

$$y_1 = A_1 e^{i(k_1x_1 - \omega_1 t_1)}$$

$$y_2 = A_2 e^{i(k_2x_2 - \omega_2 t_2)}$$

$$y_1 + y_2 = A_1 e^{i(k_1x_1 - \omega_1 t_1)} + A_2 e^{i(k_2x_2 - \omega_2 t_2)}$$

$$y_1 + y_2 = [A_1 \cos(k_1x_1 - \omega_1 t_1) + A_2 \cos(k_2x_2 - \omega_2 t_2)] + i[A_1 \sin(k_1x_1 - \omega_1 t_1) + A_2 \sin(k_2x_2 - \omega_2 t_2)$$

for wave of the same phase of different amplitude ,

$$(k_1x_1 - \omega_1 t_1) \equiv (k_2x_2 - \omega_2 t_2) = (kx - \omega t)$$

Then ,

$$y_1 = A_1 e^{i(kx - \omega t)}$$

$$y_2 = A_2 e^{i(kx - \omega t)}$$

$$y_1 + y_2 = (A_1 + A_2)\,[\cos(kx - \omega t) + i\,\sin(kx - \omega t)]$$

$$y_1 + y_2 = (A_1 + A_2)\,[\cos(kx - \omega t) + i\,\sin(kx - \omega t)]$$

HOW !!

$$|y_1 + y_2| = (A_1 + A_2)$$

For, $\boldsymbol{y_1 - y_2}$

$$y_1 - y_2 = (A_1 - A_2)\,[\cos(kx - \omega t) + i\sin(kx - \omega t)]$$

Now,

$|y_1 - y_2|$

$$|y_1 - y_2| = \sqrt{(A_1 - A_2)^2\cos^2(kx - \omega t) + (A_1 - A_2)^2\sin^2(kx - \omega t)}$$

$$|y_1 - y_2| = \sqrt{(A_1 - A_2)^2\{\underbrace{\cos^2(kx - \omega t) + \sin^2(kx - \omega t)}\}}$$

=1

Then ,

$$|y_1 - y_2| = (A_1 - A_2)$$

2.Impedance and Admittance

The ***impedance*** and ***admittance*** can be represented in rectangular or polar coordinates (Electricity; AC-circuit).

PHYSICALLY , ***impedance*** is represent by $\mathbb{Z}$

$$\mathbb{Z} = \frac{V}{I} = R + jX \quad , \quad \mathbb{Z} \angle \emptyset$$

Where, $|\mathbb{Z}| = \sqrt{R^2 + X^2}$, $\emptyset = tan^{-1}\frac{X}{R}$ " *impedance*"

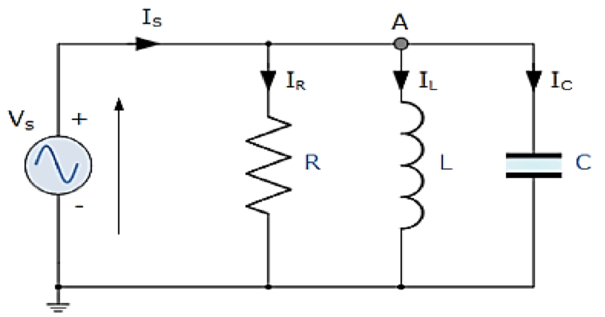
PHYSICALLY , ***admittance*** is represent by Y

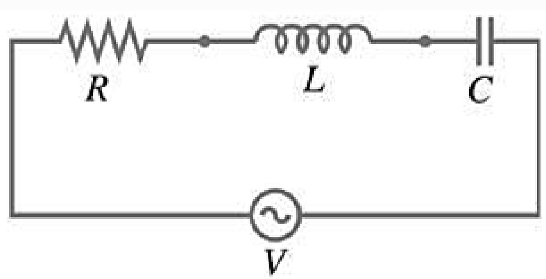
$$Y = \frac{I}{V} = G + jB = Y\angle \gamma$$

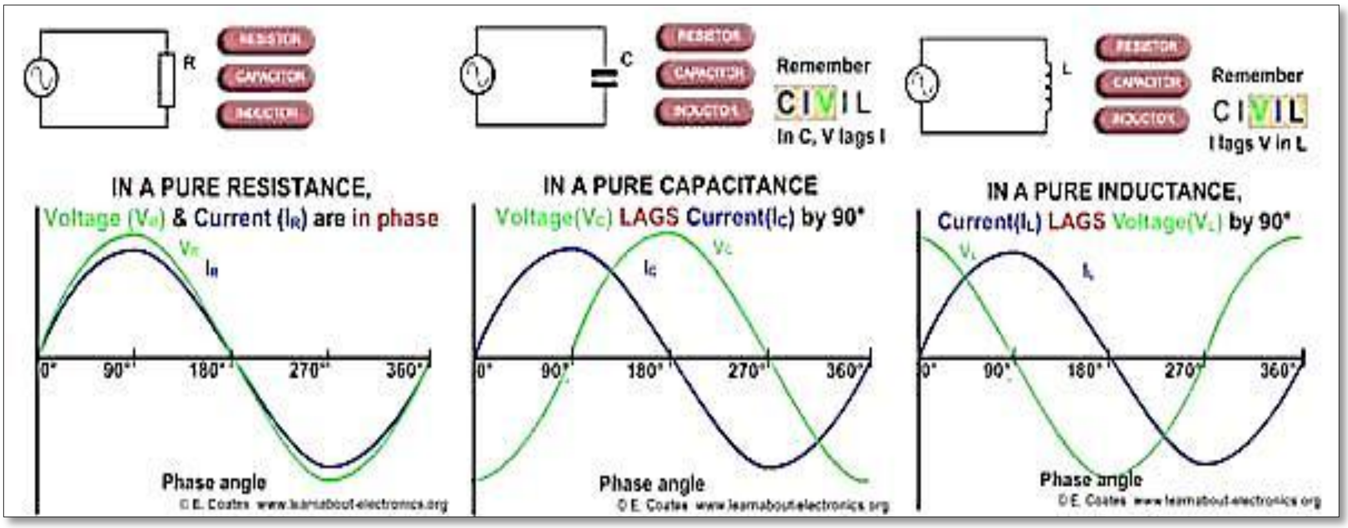
$$|Y| = \sqrt{G^2 + B^2} \quad , \quad \gamma = tan^{-1}\frac{B}{G} = -\emptyset \quad \text{" admittance "}$$

Impedances and admittances of passive elements		
Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

Parallel RLC-circuit ***Series RLC-circuit***







***Example* :**

Determine the resistance and series inductance (or capacitance) for each of the following impedences ,assuming a frequency of 50Hz:

(a) $(4.0 + j7.0)\Omega$,

(b) $-j20\Omega$,

(c) $15\angle -60^o\ \Omega$

Solution :

(a) *Impedence* , $Z = (4.0 + j7.0)\Omega$ hence , resistance $=4.0\Omega$ and reactance $= 7.0\Omega$. Since the imaginary part is *poisitive*, the reactance is *inductive* ,*i.e.* $X_L = 7.0\Omega$.

Since ,

$$X_L = 2\pi f L$$

then ***inductance*** .

$$L = \frac{X_L}{2\pi f} = \frac{7.0}{2\pi(50)} = 0.0223\ \text{H} \quad or\ 22.3\ mH$$

$\pi = 3.14$

(b) *Impedance* , $Z = -j20\Omega$, *i.e.* ,

$Z = (0 - j20)\Omega$ hence resistance = 0 and reactance $=20\Omega$.

Since the imaginaery part *is negative* , the reactance is *capacitive* , i.e. , $X_C = 20\Omega$ and since

$$X_C = \frac{1}{2\pi f C}$$

then ,*Capacitive* ,

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50)(20)} F \quad , \; C = \frac{10^6}{2\pi(50)(20)} \mu F = 159.2\ \mu F$$

(d) *Impedence* , Z=$15\angle -60^o$

$$Z=15\angle -60^o \; = 15[\cos(60^o) + jsin(-60^o)] = 7.50 - j12.99\Omega.$$

Hence,
resistance =7.50Ω and capacitivie reactance , $X_C = 12.99\Omega$.
Since

$$X_C = \frac{1}{2\pi f C} \qquad \text{then capacitive,} \; C = \frac{1}{2\pi f X_C} = \frac{10^6}{2\pi(50)(12.99)} \mu\text{F} = 245\mu\text{F}$$

Example :

An alternative voltage of 240V, 50Hz is connected across an impedance of $(60 - j100)\Omega$.

Determine the :

(a) resistance
(b) capacitance
(c) magnitude of the impedance and its phase angle and
(d) current flowing

Solution

$\pi = 3.14$

Impedance $Z = (60 - j100)\Omega$. Hence resistance = 60Ω.

(a) Capacitive reactance $X_C = 100\Omega$, and

Since

$$X_C = \frac{1}{2\pi f C}$$

then *capacitance* ,

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50)(100)} = \frac{10^6}{2\pi(50)(100)} \mu\text{F} = 31.83\ \mu\text{F}$$

(c) *Magnitude of impedance* ,

$$|Z| = \sqrt{(60)^2 + (-100)^2} = 116.6\ \mu\text{F}$$

Phase angle, or $\arg Z = tan^{-1}\left(\frac{-100}{60}\right) = -59.04^o$

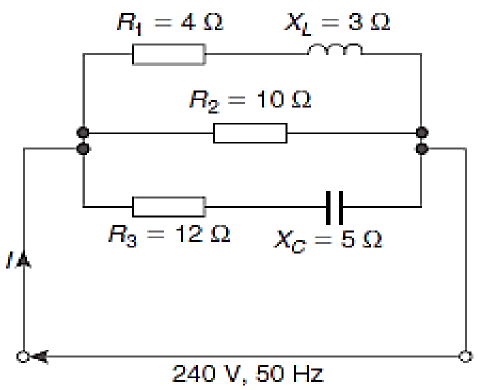
(b) *Current flowing* ,

$$I = \frac{V}{Z} = \frac{240\angle 0^o}{116.6\angle -59.04^o} = 2.058 \angle 59.04^o \text{ A}$$

Example :

For the parallel circuit shown in Fig., determine (,using complex numbers)the values of:

(a) current I ,and
(b) its phase relative to the 240V supply



Solution

- ✓ Current $I = \frac{V}{Z}$.
- ✓ Impedance Z for the three-branch parallel circuit is given by :

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

Where

$$Z_1 = 4 + j3\ , \quad Z_2 = 10 \text{ and } \quad Z_3 = 12 - j5$$

✓ Admittance , $Y_1 = \frac{1}{Z_1} = \frac{1}{4+j3}$

$$= \frac{1}{4 + j3} \times \frac{\mathbf{4 - j3}}{\mathbf{4 - j3}} = \frac{4 - j3}{4^2 + 3^2}$$

$$Y_1 = 0.160 - j0.120 \text{ siemens}$$

✓ Admittance , $Y_2 = \frac{1}{Z_2} = \frac{1}{10}$

$$Y_2 = 0.10 \text{ siemens}$$

✓ Admittance , $Y_3 = \frac{1}{3} = \frac{1}{12-j5}$

$$= \frac{1}{12-j5} \times \frac{\mathbf{12+j5}}{\mathbf{12+j5}} = \frac{12+j5}{12^2+5^2}$$

$$Y_3 = 0.0710 + j0.0296 \text{ siemens}$$

Total admittance, $\boldsymbol{Y} = \boldsymbol{Y_1} + \boldsymbol{Y_2} + \boldsymbol{Y_3}$

$= (0.160 - j0.120) + (0.10) + (0.0710 + j0.0296)$

$= 0.331 - j0.0904$

$\boldsymbol{Y} = 0.343\angle - 15.28^o$ siemens

How!!!

Current , $I = \frac{V}{Z} = VY$

سوال !! لماذا اخترنا VY ولم نختر V/Z

$I = (240\angle 0^o)\ (0.343\angle - 15.28^o) = 82.32\angle - 15.28^o\ A$

$\boldsymbol{I = 82.32\angle - 15.28^o\ A}$

3.Mechanics

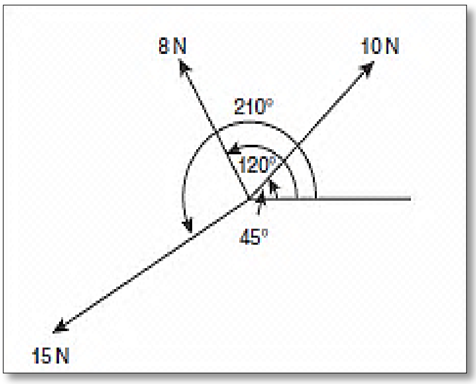
(a) ***Three coplanar forces***

Determine the

- ✓ magnitude and
- ✓ direction of the resultant of the three coplanar forces given below ,when they act at a point :
  - Force *A*, 10N acting at 45^o from the poisitive horizontal axis ,
  - Force *B* , 8N acting at $120^o$from the positive horizontal axis ,
  - Force *C* , 15N acting at $210^o$from the positive horizontal axis.

Solution

The space daigram is shown in the figure .



The forces may be written as complex numbers. Thus

force A , $F_A = 10\angle 210^o$,

force B , $F_B = 8\angle 120^o$,

force C , $F_C = 15\angle 210^o$.

- The resultant force $= f_A + f_B + f_C$

$= 10\angle 45^o + 8\angle 120^o + 15\angle 210^o$

$\boldsymbol{F} = 10(cos45^o + jsin\ 45^o) + 8(cos120^o + jsin\ 120^o) + 15(cos210^o + jsin\ 210^o)$F

$= 7.071 + j7.071) + (-4.00 + j6.928) + (-12.99 - j7.5) = -9.919 + j6.499$

$$\boldsymbol{F} = -9.919 + j6.499$$

Since

$-9.919 + j6.499$ lies in the second quadrant

- *Magnitude of resultant force*

$$F = \sqrt{(-9.919)^2 + 6.499^2} = 11.86N$$

- *Direction of resultant force*

$$= tan^{-1}\left(\frac{6.499}{-9.919}\right) = 146.77^o$$

since $-9.919 + j6.499$) lies in the second quadrant).

Solving the harmonic oscillator

A harmonic oscillator is governed by the equation

$$ma = -kx \qquad ....(1)$$

Where a is acceleration .

This provides us with the differential equation

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \qquad ... (1)$$

We know that the solution of this equation can be written as sine and cosine :

$$x(t) = Acos(\omega_0 t) + Bsin\,(\omega_0 t)$$

Where $\omega_0 = \sqrt{k/m}$, and the constants A and B are chosen to match initial conditions.

For example if $x(0) = x_0$ and $v(0) = \left(\frac{dx}{dt}\right)\Big|_{t=0}$, then

$$A = x_0 \quad , B = 0$$

An equivalent way to write this solution is to put

$$x = Ae^{i(\omega_0 t + \phi_0)})$$

And match initial conditons by adjusting the constants A and ϕ_0 . With the initial conditions chosen above .we would put

.

$$A = x_0 \text{ and } \phi_0 = 0$$

This is easy to see:

$$\frac{dx}{dt} = i\omega_0 A e^{i(\omega_0 t+\phi_0)} = i\omega_0 x \qquad \dots (2)$$

$$\frac{d^2x}{dt^2} = -\omega_0^2 A e^{i(\omega_0 t+\phi_0)} = -\omega_0^2 x \qquad \dots (3)$$

Substitute eq. (3), (2) in to (1)

$$-\omega_0^2 m x + kx = 0$$

Then the solution satisfied

$$\omega_0 = \sqrt{k/m}$$