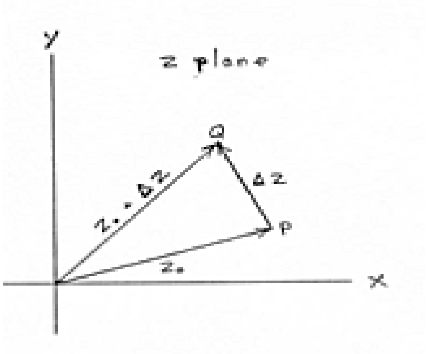
Prof. Dr. Hayfa G. Rashid

Complex Derivative

المحاضرة الثامنــة :

مشتقة الدوال العقدية وتطبيقاتها

Derivative of complex function were defined in terms of *limit* and the other in terms of *formulas* ;



أي لغرض ايجاد مشتقة الدالة العقدية فان هناك طريقتين :

- ✓ الاولى هي استخدام (الغايات) و
- ✓ الثانية استخدام (قواعدالمشتقات)

و هو نفس الاسلوب المتبع سابقا في ايجاد مشتقة الدوال في حسابان التكامل و التفاضل للدوال الحقيقية وكالاتي :

$$f'(Z_0) = \frac{\Delta w}{\Delta Z} = \lim_{\Delta Z \to 0} \frac{f(Z_0 + \Delta Z) - f(Z_0)}{\Delta Z}$$

Or,

$\frac{dw}{dZ} = f^{'}(z)$	$\mathbf{1^{st}}$ derivative of w with respect to z
$\frac{d^2w}{dZ^2} = \frac{d}{dz}(\frac{dw}{dz}) = f^{''}(z)$	$\mathbf{2^{nd}}$ derivative of w with respect to z
$\frac{d^nw}{dZ^n} == f^{(n)}(z)$	$\mathbf{n^{th}}$ derivative of w with respect to z

Rules for differentation

If $f(z), g(z)$ and $h(z)$ are " *analytic* " function of z the following differentation rules are *valid* :

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1.	$$\frac{d}{dz}[f(z)+g(z)]=\frac{d}{dz}f(z)+\frac{d}{dz}g(z)$$ $$=f'(z)+g'(z)$$
2.	$$\frac{d}{dz}[f(z)-g(z)]=\frac{d}{dz}f(z)-\frac{d}{dz}g(z)$$ $$=f'(z)-g'(z)$$
3.	$\frac{d}{dz}\{cf(z)\}=c\frac{d}{dz}f(z)$, and c is constant
4.	$$\frac{d}{dz}[f(z)g(z)]=f(z)\frac{d}{dz}g(z)+g(z)\frac{d}{dz}f(z)$$ $$=f(z)g'(z)+g(z)f'(z)$$
5.	$$\frac{d}{dz}\left\{\frac{f(z)}{g(z)}\right\}=\frac{g(z)\frac{d}{dz}f(z)-f(z)\frac{d}{dz}g(z)}{[g(z)]^2}$$ $=\frac{g(z)f'(z)-f(z)g'(z)}{[g(z)]^2}$, if $g(z)\neq 0$

والجدول التالي يوجز مشتقات اهم الدوال :

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$\frac{d}{dz}z^a = az^{a-1}$	$\frac{d}{dz}z^z = z^z(1+\ln z)$
$\frac{d}{dz}a^z = a^z\ln(a)$	$\frac{d}{dz}\ln z = \frac{1}{z}$
$\frac{d}{dz}e^z = e^z$	$\frac{d}{dz}\log_a z = \frac{1}{z\ln a}$
$\frac{d}{dz}\sin(z) = \cos(z)$	$\frac{d}{dz}\sinh(z) = \cosh(z)$
$\frac{d}{dz}\cos(z) = -\sin(z)$	$\frac{d}{dz}\cosh(z) = \sinh(z)$
$\frac{d}{dz}\tan(z) = \sec^2(z)$	$\frac{d}{dz}\tanh(z) = 1-\tanh^2(z) = \text{sech}^2(z)$
$\frac{d}{dz}\cot(z) = -\csc^2(z)$	$\frac{d}{dz}\coth(z) = -\text{csch}^2(z)$
$\frac{d}{dz}\csc(z) = -\csc(z)\cot(z)$	$\frac{d}{dz}\text{csch}(z) = -\text{csch}(z)\coth(z)$
$\frac{d}{dz}\sec(z) = \sec(z)\tan(z)$	$\frac{d}{dz}\text{sech}(z) = -\text{sech}(z)\tanh(z)$
$\frac{d}{dz}\sin^{-1}(z) = \frac{1}{\sqrt{1-z^2}}$	$\frac{d}{dz}\sinh^{-1}(z) = \frac{1}{\sqrt{1+z^2}}$
$\frac{d}{dz}\cos^{-1}(z) = -\frac{1}{\sqrt{1-z^2}}$	$\frac{d}{dz}\cosh^{-1}(z) = \frac{1}{\sqrt{z+1}\sqrt{z-1}}$
$\frac{d}{dz}\tan^{-1}(z) = \frac{1}{1+z^2}$	$\frac{d}{dz}\tanh^{-1}(z) = \frac{1}{1-z^2}$
$\frac{d}{dz}\cot^{-1}(z) = -\frac{1}{1+z^2}$	$\frac{d}{dz}\coth^{-1}(z) = \frac{1}{1-z^2}$
$\frac{d}{dz}\csc^{-1}(z) = -\frac{1}{z\sqrt{z^2-1}}$	$\frac{d}{dz}\text{csch}^{-1}(z) = -\frac{1}{z^2\sqrt{1+\frac{1}{z^2}}}$
$\frac{d}{dz}\sec^{-1}(z) = \frac{1}{z\sqrt{z^2-1}}$	$\frac{d}{dz}\text{sech}^{-1}(z) = -\frac{1}{z(z+1)\sqrt{\frac{1-z}{1+z}}}$

Example 1 : Find the 1st derivative for the function

$$f(z) = [z + (z^2+1)^2]^2$$

$$\frac{d}{dz}[z+(z^2+1)^2]^2 = 2[z+(z^2+1)^2]\frac{d}{dz}[z+(z^2+1)^2]$$
$$= 2[z+(z^2+1)^2][1+2(z^2+1)\times 2z]$$

At $z = 1+i$, then

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$$\frac{d}{dz}[z+(z^2+1)^2]^2$$

$$= 2[(1+i)+\{(1+i)^2+1\}^2]\times[1+2\{(1+i)^2+1)\times 2(i)\}]$$

$$= \dots\dots$$

Example 2 : Find the 1st derivative for the function

$$f(z) = \left[\frac{(z+2i)(i-z)}{(2z-1)}\right] \ , at \quad z = i$$

Solution

$$(z+2i)(i-z) = zi - z^2 - 2 - 2iz = -2 - iz - z^2$$

$$\frac{d}{dz}\left[\frac{(z+2i)(i-z)}{(2z-1)}\right] = \frac{d}{dz}\left[\frac{-2-iz-z^2}{(2z-1)}\right] = \frac{(2z-1)(-i-2z)-(-2-iz-z^2)(2)}{(2z-1)^2}$$

Example 3 : Find the 1st derivative for the function $[z+(z^2+1)^2]^2$

$$\frac{d}{dz}[z+(z^2+1)^2]^2 = 2\{z+(z^2+1)^2\} * [1+2(z^2+1)(2z)]$$

Example 4 : Find the 1st derivative for the function

$$f(z) = (z+2\sqrt{z})^{1/3}$$

$$\frac{d}{dz}(z+2\sqrt{z})^{1/3} = \frac{1}{3}[(z+2\sqrt{z})^{-2/3}] * \left(1+2\left(\frac{1}{2}\right)z^{-1/2}\right] = \dots\dots\dots$$

Example 5 : Find $\frac{d}{dz}(1+z^2)^{3/2}$

$$\frac{d}{dz}(1+z^2)^{\frac{3}{2}} = \frac{3}{2}(1+z^2)^{1/2} \times \frac{d}{dz}(1+z^2)$$

$$\frac{d}{dz}(1+z^2)^{3/2} = \frac{3}{2}(1+z^2)^{1/2} \times (2z) = \cdots \dots$$

Example 6 : Find $\frac{d}{dz}\left[\frac{(z+2i)(i-z^2}{(2z-1)^2}\right]$ $at \quad z = i$

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Example 7 : Find 1^{st} derivative for the function z^{lnz}

Let , $w = z^{lnz}$

$$lnw = (lnz) \times ln(z) \rightarrow \quad lnw = (ln\, z)^2$$

$$\frac{1}{w}\frac{dw}{dz} = 2ln\, z\left(\frac{1}{z}\right) \rightarrow \frac{dw}{dz} = w \times 2ln\, z \times \left(\frac{1}{z}\right)$$

$$\therefore \frac{dw}{dz} = z^{lnz} \times 2lnz \times z^{-1}$$

$$\frac{dw}{dz} = 2(z^{lnz-1})lnz$$

Example 8 : Find $\frac{dw}{dz}$ for the function z^i

Let , $w = z^i$

$$lnw = ln(z^i) \rightarrow \quad lnw = i(lnz)$$

$$\frac{1}{w}\frac{dw}{dz} = i\left(\frac{1}{z}\right) \rightarrow \frac{dw}{dz} = i\left(\frac{1}{z}\right) * w$$

$$\therefore \frac{dw}{dz} = w \times \frac{i}{z} = \left(\frac{i}{z}\right)(z^i) = iz^{i-1}$$

Example 9 : Find $\frac{dw}{dz}$ $\{sin(iz-2)^{tan^{-1}(z+3i)}\}$

Let , $w = sin(iz-2)^{tan^{-1}(z+3i)}$

$$lnw = tan^{-1}(z+3i) \times ln\{sin(iz-2)\}$$

$\frac{1}{w}\frac{dw}{dz} = tan^{-1}(z+3i) \times \frac{1}{sin(iz-2)} \times cos(iz-2) \times i + ln\{sin(iz-2) \times \frac{1}{1+(z+3i)^2} \times (1)\}$

$$\frac{1}{w}\frac{dw}{dz} = w \times \dots\dots$$

Continue

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Example 10 : Find $\frac{d}{dz}\, ln[z-\frac{3}{2}+\sqrt{z^2-3z+2i}\,]$

$$\frac{d}{dz} ln\left[z-\frac{3}{2}+\sqrt{z^2-3z+2i}\right]=\frac{1}{z-\frac{3}{2}+\sqrt{z^2-3z+2i}}\;\frac{d}{dz}[z-\frac{3}{2}+\sqrt{z^2-3z+2i}]$$

$$=\frac{1}{z-\frac{3}{2}+\sqrt{z^2-3z+2i}}\;[1+\frac{1}{2}(z^2-3z+2i)^{-1/2}(2z-3)]$$

Example 11 : Find $\frac{d}{dz}\, tan^{-1}(z+3i)^{-1/2}$

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$$\frac{d}{dz}\, tan^{-1}(z+3i)^{-1/2}=\frac{1}{1+[(z+3i)^{-1/2}]^2}\times\frac{d}{dz}(z+3i)^{-1/2}$$

$$=\frac{1}{1+\left(\frac{1}{\sqrt{z+3i}}\right)^2}(-\frac{1}{2})(z+3i)^{-\frac{3}{2}}\times(1)$$

Continue

Example 12 : Find $\frac{d}{dz}\, cos^{-1}(sinz-cosz)$

$$\frac{d}{dz}\, cos^{-1}(sinz-cosz)=\frac{-1}{\sqrt{1-(sinz-cosz)^2}}\;\frac{d}{dz}\,(sinz-cosz)$$

Example 13 : Find $\frac{d}{dz}\, sinh(z+1)^2$

$$\frac{d}{dz}\, sinh(z+1)^2=cosh(z+1)^2\times 2(z+1)(1)$$

Example 14 : Find $\frac{d}{dz}\, cosh^{-1}(lnz)$

$$\frac{d}{dz}\, cosh^{-1}(lnz)=-\frac{1}{\sqrt{1-(lnz)^2}}\times\frac{1}{z}$$

Activity : Find $\frac{d}{dz}\,5^{(z^2+i)}$, *Hint* $\frac{d}{dz}\,a^z=a^z\times lna\times\frac{d}{dz}(z)$

ثم قارن مع مثال7 ، 8 و 9 ماذا تستنتج !!!

$$\frac{d}{dz}\,5^{(z^2+i)}=5^{(z^2+i)}\times ln5\times\frac{d}{dz}(z^2+i)=5^{(z^2+i)}\times ln5\times(2z)$$