Syllabus lecture 5 chapter 4 – thin lenses

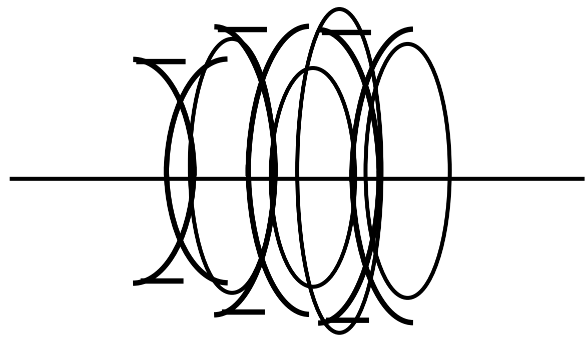
Thin-lens combinations, The power of a thin lens, Thick Lenses, two spherical surfaces. Focal points and principal points, General Thick- Lens Formula,

cardinal points, problem

Combination of Thin Lenses

Suppose that we have in general a system of N lenses whose thicknesses are small and each lens is placed in contact with its neighbor

$$\frac{1}{f_{ef}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \cdots \frac{1}{f_N}$$



a) **If the optical system is consisting of two lenses is contact**

The effective focal length of combination of two lenses f1 and f2 is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

The power of two lensesP1 and P2 or more ,placed in contact is the sum of two lenses

P=P1+ P2

Example (3): A meniscus(concave-convex)lens has an index of refraction 1.5 and the radii of curvature of its surface are 10 and 20 cm .the concave surface is upward and it is filled with an oil of refractive index 1.6 .calculate the focal length of oil-glass combination

Solution:

For the glass

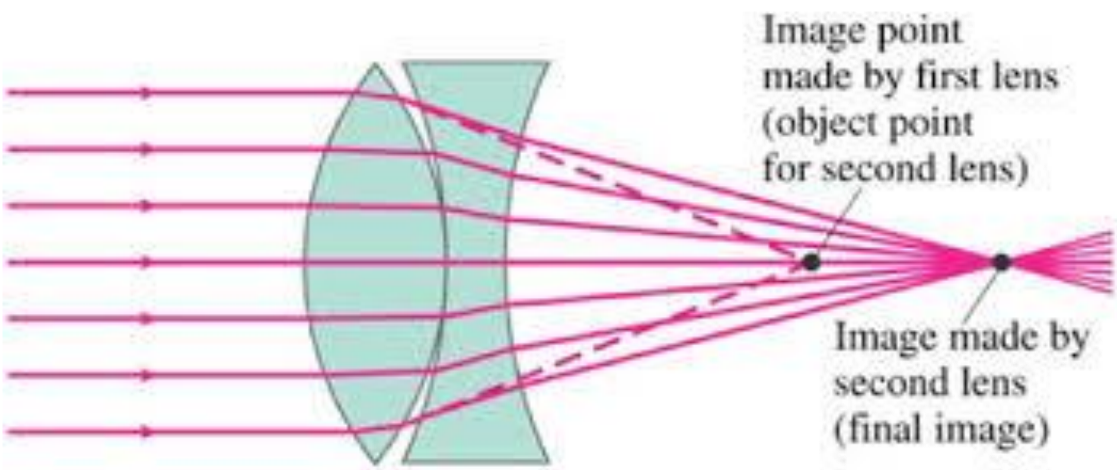
$$\frac{1}{f_1} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= (1.5-1)\left(\frac{1}{10\ cm} - \frac{1}{20\ cm}\right) = \frac{1 \quad 1}{40\ cm f_1}$$

$$= (1.6-1)\left(\frac{1}{20\ cm} - \frac{1}{\infty}\right) = \frac{3 \quad 1}{100\ cm f_2}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{40\ cm} + \frac{3}{100\ cm} = \frac{11}{200\ cm}$$

$$\therefore f = 18.2\ m$$



2) Two lens combination separated by distance d

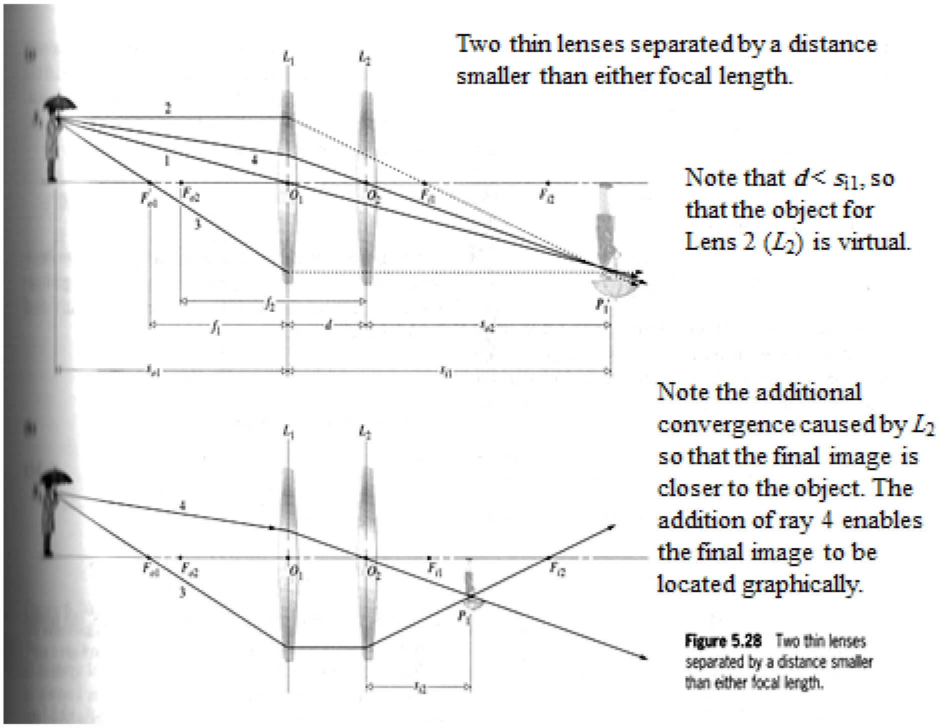


Figure 5.28 Two thin lenses separated by a distance smaller than either focal length.

a) Note that $d < s_{i1}$, so that the object for Lens 2 (L_2) is virtual

Solution:
Step 1 :

$$\frac{1}{s_1} + \frac{1}{s'_{1_1}} = \frac{1}{f_1}$$

$=\frac{f_1 x s_1}{s_1 - f_1} s'_{i_1}$

2) step 2 $\qquad s_{o2} = d - s'_{i1}$

3) $s''_{i_2} = \frac{f_2 x\, s_{o2}}{s_{o2} - f_2} \quad the\ final\ image$

4) $m_{t=} m_1 m_2$ the size of final image

5) The focal length effective equation of two lenses separated at distance d or apart between them

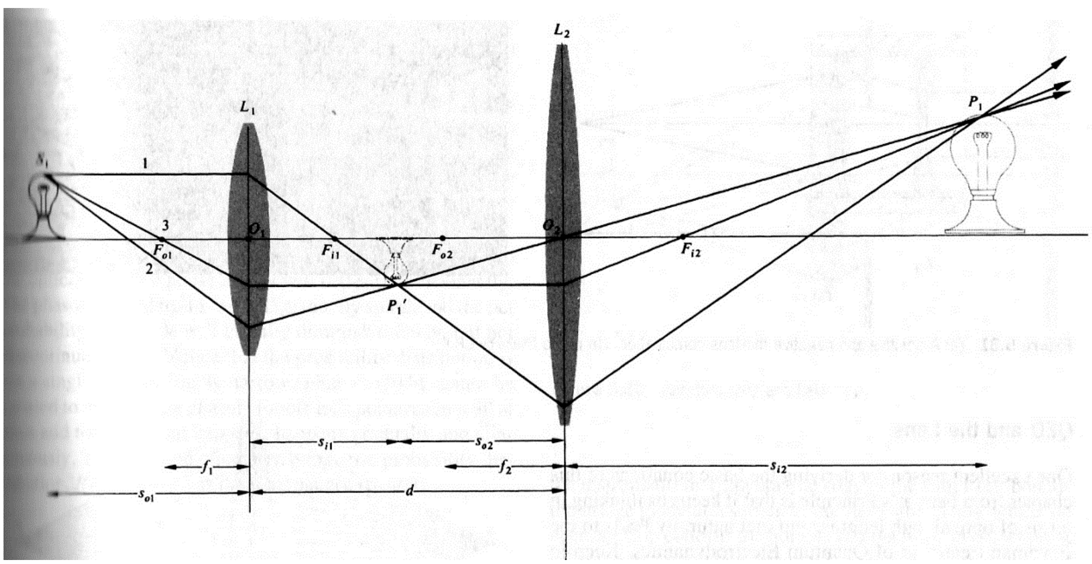
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

6) Power

When two lenses of focal lenses f_1 and f_2 are placed coaxial and separated by a distance d the combined power

$$P = P_1 + P_2 - d\, P_1 P_2$$

3. Note that $d > s_{i1}$, so that the object for Lens 2 (L_2) is real



Example 8: A compound lens consists of two thin-bi-convex lenses L_1 and L_2 of focal lengths 10 cm and 20 cm , separated by a distance of 80 cm. describe the image corresponding to a 5 cm tall object 15 cm from the first lens.

Solution:

$$\frac{1}{s_{i1}}=\frac{1}{f_1}-\frac{1}{s_{o1}}, \quad s_{i1}=\frac{s_{o1}f_1}{s_{o1}-f_1}$$

$$s_{o2}=\ddot{d}-s_{i1} \qquad \begin{array}{l} s_{o2}<0 \ (virtual) \\ s_{o2}>0 \ (real) \end{array}$$

$$\frac{1}{s_{i2}}=\frac{1}{f_2}-\frac{1}{s_{o2}}, \quad s_{i2}=\frac{s_{o2}f_2}{s_{o2}-f_2}=\frac{(d-s_{i1})f_2}{(d-s_{i1}-f_2)}=\frac{f_2 d-\frac{f_2 s_{o1}f_1}{s_{o1}-f_1}}{d-f_2-\frac{s_{o1}f_1}{s_{o1}-f_1}}$$

s02= 15 cm, d=80 cm, f=20 cm , f1= 10 cm

$$s_{i2} = \frac{f_2\, d \;-\; [\frac{f_1 f_2 \quad s_o}{(s_o \; - f_1)}]}{d - f_2 \;-[f_1 s_o \;/(\,s_{o-}\;f_1)]}$$

$$s_{i2} = \frac{20x80 \;-\; [\frac{10x20x15}{(10x15)}]}{80-20-[\frac{(10x15)}{15-10}]}$$

$$=\frac{100}{3} \quad = \quad 33.3\ cm$$

$$M_T = M_{T1}M_{T2} = \left(-\frac{s_{i1}}{s_{o1}}\right)\left(-\frac{s_{i2}}{s_{o2}}\right) = \frac{f_1 s_{i2}}{d(s_{o1}-f_1)-s_{o1}f_1}$$

4) For this two lens system, let’s determine the front focal length (*ffl*) f_1 and the back focal length (*bfl*) f_2.

Let $s_{i2} \to \infty$ then this gives $s_{o2} \to f_2$.

$s_{o2} = d - s_{i1} = f_2 \Rightarrow s_{i1} = d - f_2$ but

$$\left[\frac{1}{S_{o1}}\right]_{s_{i2\to\infty}} = \frac{1}{f_1} - \frac{1}{s_{i1}} = \frac{1}{f_1} - \frac{1}{d - f_2} \quad \Rightarrow \quad ffl = \left[s_{o1}\right]_{s_{i2\to\infty}} = \frac{f_1(d - f_2)}{d - (f_1 + f_2)}$$

From the previous examples, we calculated s_{i2}. Therefore, if $s_{o1} \to \infty$

we get,

Two thin lenses separated by a distance smaller than either focal length

Note that $d < s_{i1}$, so that the object for Lens 2 (L_2) is virtual

$$bfl = s_{i2} = \frac{f_2 d - f_2 f_1}{d - f_2 - f_1} = \frac{f_2(d - f_1)}{d - (f_2 + f_1)}$$

$$for \quad d \to 0, \quad bfl = ffl = \frac{f_2 f_1}{f_2 + f_1} = f_{ef}$$

$$\Rightarrow \quad \frac{1}{f_{ef}} = \frac{1}{f_1} + \frac{1}{f_2}$$

Let $d \to 0$ (this is the thin lens approximation) and $n_m \approx 1$:

$$\Rightarrow \quad \frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (c)$$

and is known as the **thin-lens equation**, or the **Lens maker's formula**, in which $s_{o1} = s_o$ and $s_{i2} = s_i$, $V_1 \to V_2$, and $d \to 0$. Also note that

$$\lim_{s_o \to \infty} s_i = f_i, \qquad \lim_{s_i \to \infty} s_o = f_o$$

For a thin lens (c) $\to f_i = f_o = f$ and

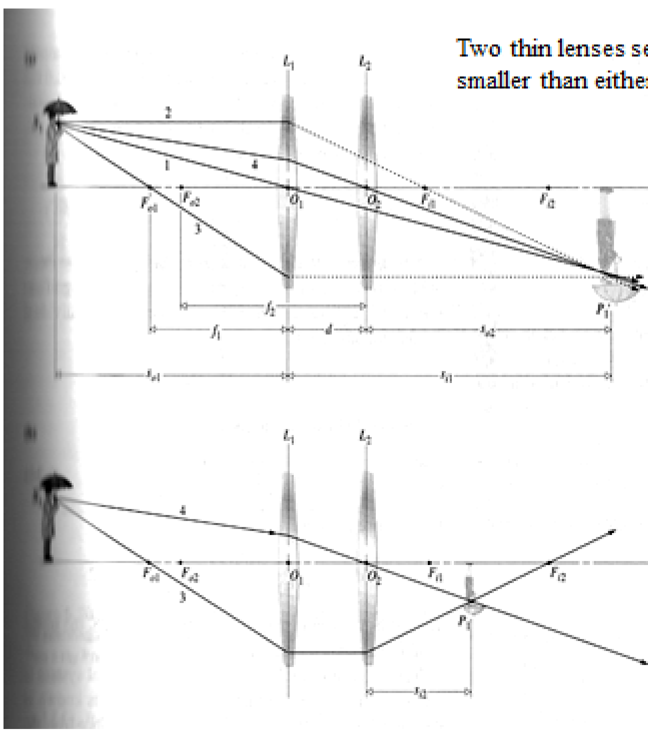
$$\frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Convex $\Rightarrow f > 0$

Concave $\Rightarrow f < 0$

$$\Rightarrow \quad \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Also, known as the Gaussian lens formula



Two thin lenses separated by a distance smaller than either focal length.

Note that $d < s_{i1}$, so that the object for Lens 2 (L_2) is virtual.

Note the additional convergence caused by L_2 so that the final image is closer to the object. The addition of ray 4 enables the final image to be located graphically.

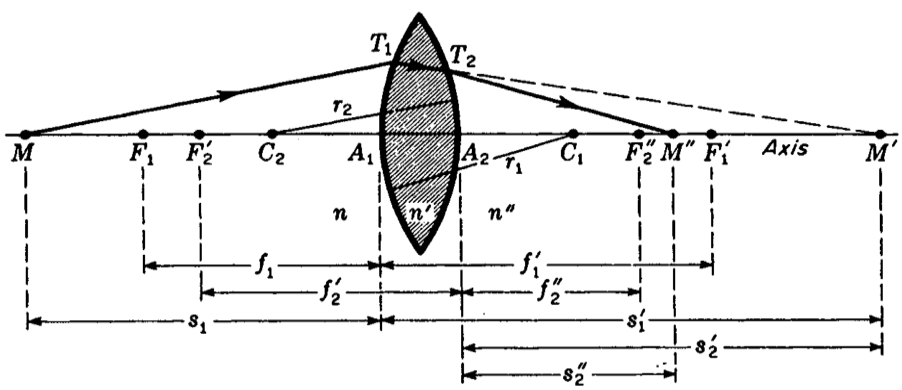
Figure 5.28 Two thin lenses separated by a distance smaller than either focal length.

TWO SPHERICAL SURFACES

formed by a thick lens and then apply the general formulas already given for calculating image distances.The formulas to be used are

$$\frac{n}{s_1} + \frac{n'}{s_1'} = \frac{n' - n}{r_1} \qquad \frac{n'}{s_2'} + \frac{n''}{s_2''} = \frac{n'' - n'}{r_2}$$

For first surface *For second surface*



Thick lens: thick lens is a physically large lens having two spherical surfaces separated by a distance , the position and size of the image can be determined directly if we know the cardinal points of the thick lens.

Let *n*, *n'*, and *n''* represent the refractive indices of three media separated by two spherical surfaces of radius r_1 and r_2 . A light ray from an axial object

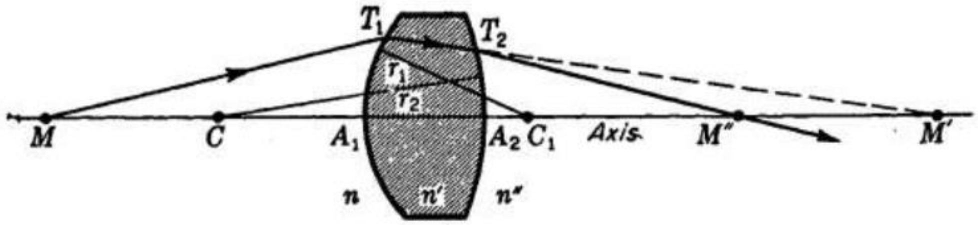


Figure 5.1: Details of the refraction of a ray at both surfaces of a lens.

We shall first consider the parallel-ray method for graphically locating an image formed by a thick lens and then apply the general formulas already given for calculating image distances. The formulas to be used are

$$\frac{n}{s_1} + \frac{n'}{s_1'} = \frac{n' - n}{r_1} \qquad \frac{n'}{s_2'} + \frac{n''}{s_2''} = \frac{n'' - n'}{r_2}$$

For first surface *For second surface*

Refraction by a thick lens

The general problem of refraction by a thick lens is solved by applying the equation for the refraction at one surface to each surface in turn. The image formed by the first surface is treated as an object for the second surface. By using this approach, you can easily obtain the formulae for the focal distances *f* for both hemisphere and sphere in terms of *R* and *n*. As an alternative, you can try to derive these equations using ray tracing. Assume that the angles are small so that $\sin\theta \approx \theta$

The **first focal point** of a lens may be defined as the object point on the lens axis which is imaged by the lens at infinity. Rays diverging from the first focal point are parallel to the axis of the lens after refraction (Figure 2(a))

The

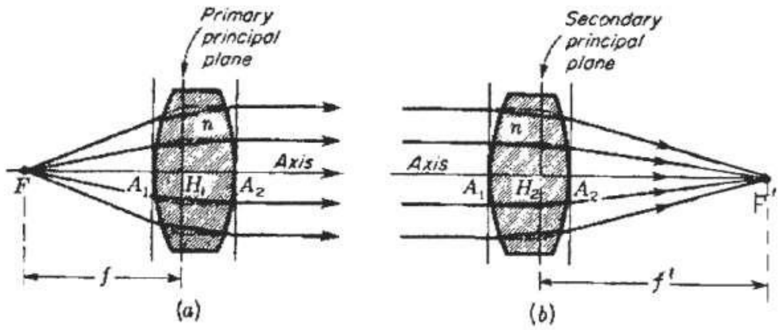


Figure 2. Primary and secondary principal planes of a thick lens. A_1 and A_2 are the vertexes.

The **second focal point** of a lens may be defined as the image point of an infinitely distant point object on the axis lens. Rays incident on the lens, parallel to the lens axis, pass through the second focal point after the refraction (Figure 2 (b)).

Computation of the position of the image of a given object at a given distance from the thick lens is facilitated by determining the positions of two planes inside the lens, known as the **principal planes.** They are defined as follows:

- First principal plane: For a cone of rays diverging from the first focal point, when the incident and emergent rays are projected ahead and back, their points of intersection lie in a common plane known as the **first (or primary)** principal plane (Figure 2(a)) . The intersection of the first principal plane and the axis of the lens is **the first principal point** *H1*. The two deviations at the two surfaces are equivalent to a single deviation in the first principal plane.

- Second principal plane: For a bundle of rays incident on a lens parallel to its axis, when the incident and emergent rays are projected ahead and back, the points of intersection lie in a common plane known as the **second (or secondary) principal plane** (Figure (b))**.** The intersection of the second principal plane and the axis of the lens is **the second principal point** *H2*.

The distance from the first focal point *F* to the first principal point *H1* is the first focal distance *f*, and the distance from the second principal point *H2* to the second focal point *F* is the second focal distance f'. However, if the medium on both sides of the lens has the same index of refraction, which is the case if the lens is in air, the two focal lengths are equal.

The equation derived for a thin lens and relating two conjugated points is:

While applying this equation to the thick lens, remember

that the measurable values are the distances **from the object to the**

vertex and from the image to the vertex (see Figure 3, below)**.**

A set of formulas that can be used for the calculation of important constants generally associated with a thick lens is presented below in the form of two equivalent sets.

Gaussian formulas	*Power formulas*
$\frac{n}{f} = \frac{n'}{f'_1} + \frac{n''}{f''_2} - \frac{dn''}{f'_1 f''_2} = \frac{n''}{f''}$	$P = P_1 + P_2 - \frac{d}{n'} P_1 P_2$
$A_1F = -f\left(1 - \frac{d}{f'_2}\right)$	$A_1F = -\frac{n}{P}\left(1 - \frac{d}{n'} P_2\right)$
$A_1H = +f\frac{d}{f'_2}$	$A_1H = +\frac{n}{P}\frac{d}{n'} P_2$
$A_2F'' = +f''\left(1 - \frac{d}{f'_1}\right)$	$A_2F'' = +\frac{n''}{P}\left(1 - \frac{d}{n'} P_1\right)$
$A_2H'' = -f''\frac{d}{f'_1}$	$A_2H'' = -\frac{n''}{P}\frac{d}{n'} P_1$

The Lens Maker's Equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)t}{nR_1R_2}\right)$$

The focal distance of a ball lens (the sphere) is given by:

$$f_{sphere} = \frac{nR}{2(n-1)}$$

A_1H_1 distance: $$h_1 = \frac{f(n-1)t}{nR_2}$$

H_2A_2 distance:

$$h_2 = \frac{f(n-1)t}{nR_1}$$

where R1 and R2 are the radii of the first and the second surfaces, h1 and h2 are the distances from the principal planes to the corresponding vertexes (see Figure 3), and t = A1A2 is the axial thickness of the lens (the distance between the vertexes).

3) **Nodal points**: are points on the principle axis of the lens where light

rays, without refraction intersect the optic axis.
N and *N''* and the transverse planes through them are called the **nodal planes**. This third pair of points and their associated planes are shown in figure 4.12. If the ray is to emerge parallel to its original direction, the two surface elements of the lens where it enters and leaves must be parallel to each other so that the effect is like that of a ***plane-parallel plate.*** A line between these two points crosses the axis at the ***optical center*** **C.**

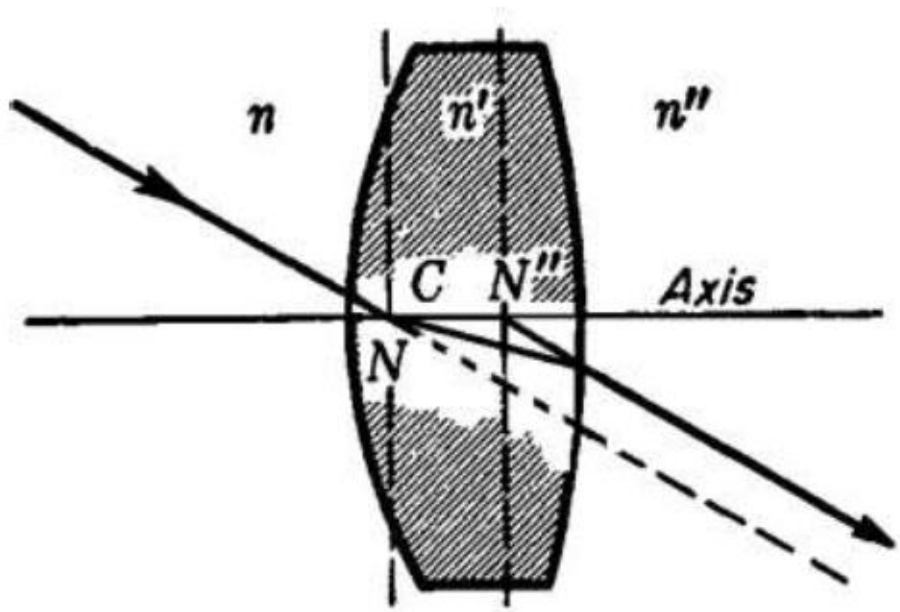


Figure (5.4): The significance of the nodal points and nodal planes of a thick lens.

EXAMPLE I An equiconvex lens 2 cm thick and having radii of curvature of 2 cm is mounted in the end of a water tank. An object in air is placed on the axis of the lens 5 cm from its vertex. Find the position of the final image. Assume refractive indices of 1.00, 1.50, and 1.33 for air, glass, and water, respectively.

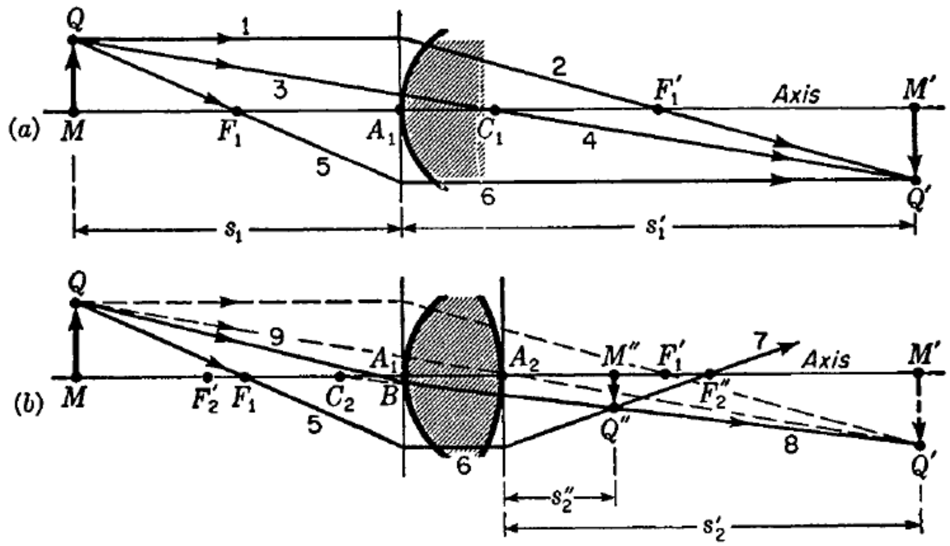


FIGURE 5B
The parallel-ray method for graphically locating the image formed by a thick lens.

$$\frac{1.00}{5} + \frac{1.50}{s_1'} = \frac{1.50 - 1.00}{2} \qquad \text{or} \qquad s_1' = +30 \text{ cm}$$

When the same equation is applied to the second surface, we note that the object distance is s_1' minus the lens thickness, or 28 cm, and that since it pertains to a virtual object it has a negative sign. The substitutions to be made are, therefore, $s_2 = -28$ cm, $n' = 1.50$, $n'' = 1.33$, and $r_2 = -2.0$ cm.

$$\frac{1.50}{-28} + \frac{1.33}{s_2''} = \frac{1.33 - 1.50}{-2} \qquad \text{or} \qquad s_2'' = +9.6 \text{ cm}$$

EXAMPLE 2 A lens has the following specifications: $r_1 = +1.5$ cm, $r_2 = +1.5$ cm, $d = 2.0$ cm, $n = 1.00$, $n' = 1.60$, and $n'' = 1.30$. Find (*a*) the primary and secondary focal lengths of the separate surfaces, (*b*) the primary and secondary focal lengths of the system, and (*c*) the primary and secondary principal points.

SOLUTION (*a*) To apply the gaussian formulas, we first calculate the individual focal lengths of the surfaces by means of Eq. (5f).

$$\frac{n}{f_1} = \frac{n' - n}{r_1} = \frac{1.60 - 1.00}{1.5} = 0.400 \qquad f_1 = \frac{1.00}{0.40} = +2.50 \text{ cm}$$

$$f_1' = \frac{1.60}{0.40} = +4.00 \text{ cm}$$

$$\frac{n'}{f_2'} = \frac{n'' - n'}{r_2} = \frac{1.30 - 1.60}{1.5} = -0.200 \qquad f_2' = \frac{1.60}{-0.20} = -8.00 \text{ cm}$$

$$f_2'' = \frac{1.30}{-0.20} = -6.50 \text{ cm}$$

(*b*) The focal lengths of the system are calculated from Eq.

$$\frac{n}{f} = \frac{n'}{f_1'} + \frac{n''}{f_2''} - \frac{d}{f_1'}\frac{n''}{f_2''} = \frac{1.60}{4.00} + \frac{1.30}{-6.50} - \frac{2.00}{4.00}\frac{1.30}{-6.50}$$

$$\frac{n}{f} = 0.40 - 0.20 + 0.10 = 0.30$$

or $f = \dfrac{1.00}{0.30} = +3.333$ cm and $f'' = \dfrac{n''}{0.30} = \dfrac{1.30}{0.30} = +4.333$ cm

The focal points of the system are given by Eqs. (5h) and (5j).

$$A_1F = -f\left(1 - \frac{d}{f_2'}\right) = -3.333\left(1 - \frac{2.0}{-8.0}\right) = -4.166 \text{ cm}$$

$$A_2F'' = +f''\left(1 - \frac{d}{f_1'}\right) = +4.33\left(1 - \frac{2.0}{4.0}\right) = +2.167 \text{ cm}$$

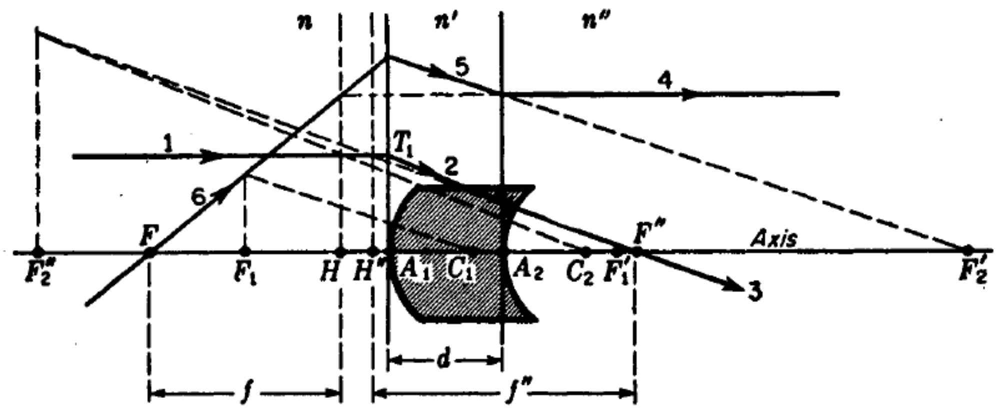


FIGURE 5H
A graphical construction for locating the focal points and principal points of a thick lens.

(*c*) The principal points are given by Eqs. (5i) and (5k).

$$A_1H = +f\frac{d}{f_2'} = +3.33\,\frac{2.0}{-8.0} = -0.833\text{ cm}$$

$$A_2H'' = -f''\frac{d}{f_1'} = -4.33\,\frac{2.0}{4.0} = -2.167\text{ cm}$$

EXAMPLE 3 Find the nodal points of the thick lens given in Example 2.

SOLUTION

To locate the primary nodal point *N,* we may use Eq. (5r) and substitute the given values of n = 1.00 and n'' = 1.30 and the already calculated value *of* f'' = +4.333 cm,

$$HN = 4.333\,\frac{1.30 - 1.00}{1.30} = +1.00\text{ cm}$$

Hence the nodal points Nand *N''* are 1.00cm to the right of their respective principal points Hand *H''.*

PROBLEMS

5.1 An equiconvex lens located in air has radii of 5.20 cm, an index of 1.680, and a thickness of 3.50 cm. Calculate *(a)* the focal length and *(b)* the power of the lens. Find *(c)* the distances from the vertices to the focal points and *(d)* the principal points.
Ans. (a) +4.43 cm, *(b)* +22.59 D, *(c)* A_1F = -3.222 cm, and A_2r = +3.222 cm, *(d)* A_1H = + 1.206 cm, and A_2H'' = -1.206 cm

5.3 A plano-convex lens 2.80 cm thick is made of glass of index 1.530. If the second surface has a radius of 3.50 cm, find *(a)* the focal length of the lens and *(b)* the power of the lens. Find the distances from the vertices to *(c)* the focal points and *(d)* the principal points.

5.5 A glass lens with radii r_l = +2.50 cm and r_2 = +4.50 cm has a thickness of 2.90 cm and an index of 1.630. Calculate *(a)* the focal length and *(b)* the power of the lens.
Find the distances from the vertices to *(c)* the focal points and *(d)* the principal points.
Ans. (a) +5.73cm,(b) *+17.46D,(e)*A_1F= -7.163 cm, and A_2F = +3.162cm,
(d) A_1H = -1.433 cm, and A_2H'' = - 2.568 cm

5.7 A glass lens with radii r_l = + 6.50 cm and r_2 = + 3.20 cm has a thickness of 2.80 cm and an index of 1.560. Calculate *(a)* the focal length and *(b)* the power of this lens in air. Find the distances from the vertices to *(c)* the focal points and *(d)* the principal points.

5.9 A thick lens with radii'l = - 4.50 cm and '2 = - 3.60 cm has a thickness of 3.0 cm and an index of 1.560. Calculate *(a)* the focal length and *(b)* the power of the lens.
Find also the distances from the vertices to the corresponding *(c)* focal points and *(d)* principal points.
Ans. (a) + 14.64 cm, *(b)* +6.83 D, *(c) A1F* = -10.26 cm, and *A2P''* = + 18.14 cm,
(d) A1H =