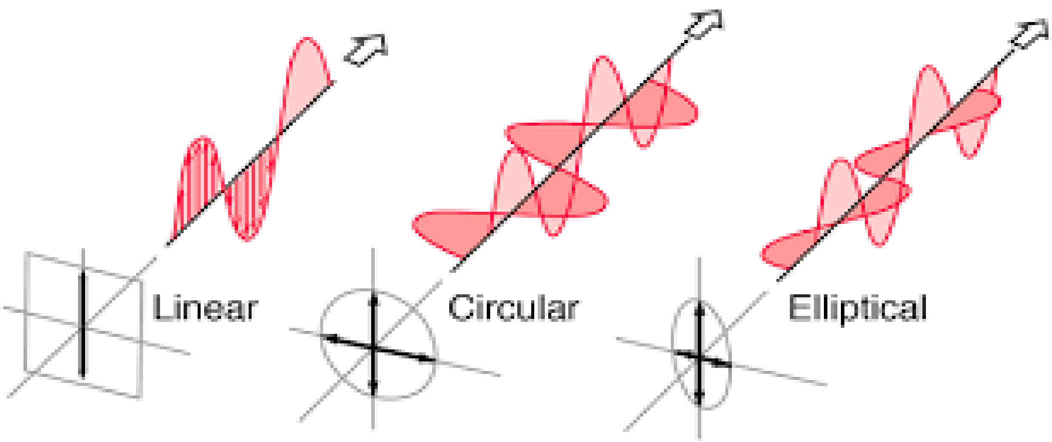
Ch. 7 Polarization)

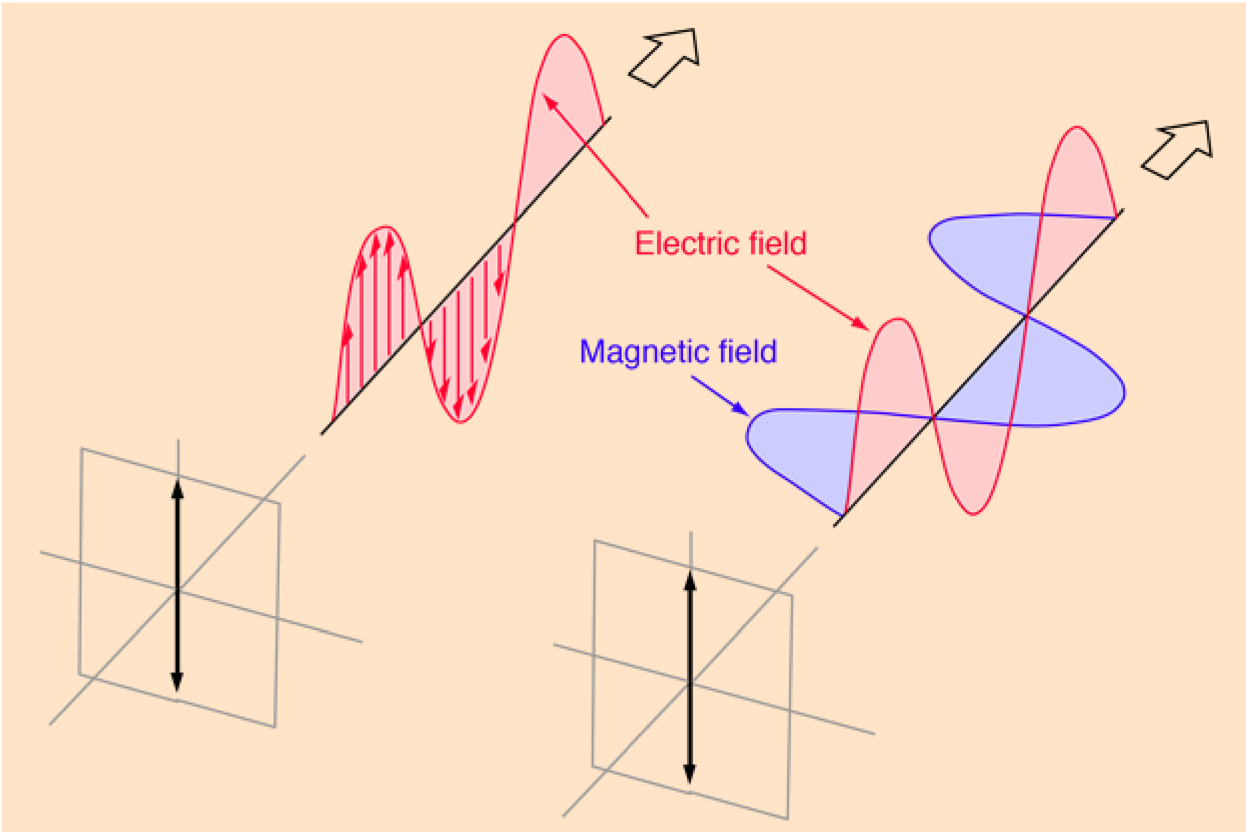
1- Plane (linear) polarization
2- Circular polarization
3- Elliptical polarization
4- Jones vector
5- Description of L,C and E polarization using Jones vector
6- Jones vector for circular polarization
7- Jones vector for elliptical polarization
8- Degree of polarization
9- Linear polarization (Jones matrix)
10- Phase retarder
11- Rotator

1- Types of polarization

a) linearly polarized



A plane electromagnetic wave is said to be linearly polarized. The transverse electric field wave is accompanied by a magnetic field wave as illustrated.



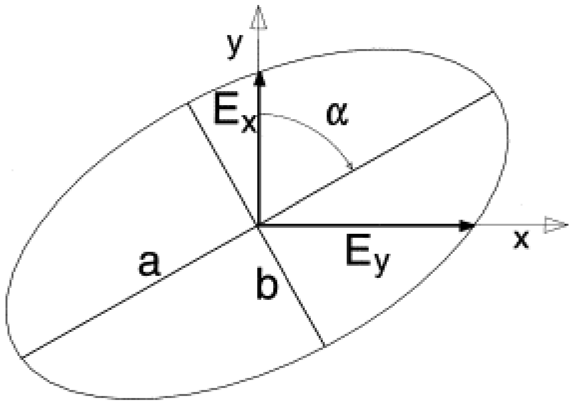
b) Circularly polarized light consists of two perpendicular electromagnetic plane waves of equal amplitude and 90° difference in phase. The light illustrated is right- circularly polarized.

f light is composed of two plane waves of equal amplitude but differing in phase by 90°, then the light is said to be circularly polarized.

c) elliptical Polarization

For elliptically polarized light the electric field vector rotates at ω but varies in amplitude so that the tip traces out an ellipse in time at a fixed position *z*. Elliptical polarization is the most general state and linear and circular polarizations are simply special degenerate forms of elliptically polarized light. Because of this generality, attributes of this state can be applied to all polarization states.

The polarization ellipse (Fig. 4) can provide useful quantities for describing the polarization state. The azimuthal angle α of the semi-major ellipse axis from the *x* axis is given by



ولكنه يختلف في السعة بحيث يتتبع الطرف القطع الناقص في ω بالنسبة للضوء المستقطب بيضاويًا ، يدور متجه المجال الكهربائي عند الاستقطاب الإهليلجي هو الحالة الأكثر عمومية ، والاستقطابات الخطية والدائرية هي ببساطة أشكال .z الوقت المناسب في موضع ثابت يمكن أن .بسبب هذه العمومية ، يمكن تطبيق سمات هذه الحالة على جميع حالات الاستقطاب .منحطة خاصة للضوء المستقطب إهليلجي لمحور القطع الناقص α يتم إعطاء الزاوية السمتي .يوفر القطع الناقص للاستقطاب (الشكل 4) كميات مفيدة لوصف حالة الاستقطاب بواسطة x شبه الرئيسي من المحور

2- Mathematical representation of Polarizer: Jones Matrix

Introducing the space and time dependence of the component vibrations,

$$E_x = E_{0x} e^{i(kz-\omega t+\varphi_x)} \quad (14\text{-}2)$$

and

$$E_y = E_{0y} e^{i(kz-\omega t+\varphi_y)} \quad (14\text{-}3)$$

for component waves traveling in the $+z$-direction with amplitudes E_{0x} and E_{0y} and phases φ_x and φ_y. Combining with Eq. (14-1),

$$\mathbf{E} = \mathbf{i}E_{0x}e^{i(kz-\omega t+\varphi_x)} + \mathbf{j}E_{0y}e^{i(kz-\omega t+\varphi_y)}$$

which may also be written

$$\mathbf{E} = [\mathbf{i}E_{0x}e^{i\varphi_x} + \mathbf{j}E_{0y}e^{i\varphi_y}]\, e^{i(kz-\omega t)} = \tilde{\mathbf{E}}_0 e^{i(kz-\omega t)} \quad (14\text{-}4)$$

The bracketed quantity, separated into x-and y-components, is now recognized as the complex amplitude $\tilde{\mathbf{E}}_0$ for the polarized wave. Since the state of polarization of the light is completely determined by the relative amplitudes and phases of these components, we need concentrate only on the complex amplitude, written as a two-element matrix, or *Jones vector*,

$$\tan(\epsilon) = \tan\left[\sin^{-1}(\sin 2\beta \sin\Delta\varphi)/2\right].$$

Polarization is right-elliptical when $0° < \Delta\phi < 180°$ and $\tan(\epsilon) > 0°$ and left-elliptical when $-180° < \Delta\phi < 0°$ and $\tan(\epsilon) < 0°$.

Jones vector

$$\tilde{\mathbf{E}}_0 = \begin{bmatrix} \tilde{E}_{0x} \\ \tilde{E}_{0y} \end{bmatrix} = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}e^{i\varphi_y} \end{bmatrix}$$

Let us determine the particular forms for Jones vectors that describe linear. circular. elliptical

Polarization

[n this case we set Eox=o, Eoy =1

The Jones vector for vertically linearly,

$$E_0 = \begin{bmatrix} 0 \\ A \end{bmatrix} = A\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Similarly, Figure 14-2b represents horizontally polarized light, for which, letting $E_{0y} = 0$, $\varphi_x = 0$, and $E_{0x} = A$,

$$E_0 = \begin{bmatrix} A \\ 0 \end{bmatrix} = A\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

the case that reduces to the vertically polarized when α = 90° and to the horizontal polarized mode when α = 0°.

$$\tilde{E}_0 = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}e^{i\varphi_y} \end{bmatrix} = \begin{bmatrix} A \cos \alpha \\ A \sin \alpha \end{bmatrix} = A\begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix}$$

Example: the jones vector for α =6 0°

$$E_0 = \begin{bmatrix} \cos 60 \\ \sin 60 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$$

Thus the Jones vector (l/V2)[J] circularly polarized light when E rotates counterclockwise, viewed head-on. This mode is called left-circularly polarized light or right circularly polarized light is:

$$E_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

It is also possible to produce elliptically polarized light with principal axes inclined to the xy-axes, as evident in Figure 14-4. This situation occurs when the phase difference between component vibrations is some angle other than $m\pi$ (linear polarization) or $(m + \frac{1}{2})\pi$ (circular or elliptical polarization oriented symmetrically about the xy-axes). Here $m = 0, 1, 2, \ldots$. For example, consider the case where E_x leads E_y by some angle ϵ, that is, $\varphi_y - \varphi_x = \epsilon$. Taking $\varphi_x = 0$, $\varphi_y = \epsilon$, $E_{0x} = A$, and $E_{0y} = b$, the Jones vector is

$$\tilde{E}_0 = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}e^{i\varphi_y} \end{bmatrix} = \begin{bmatrix} A \\ be^{i\epsilon} \end{bmatrix}$$

Using Euler's theorem we write

$$be^{i\epsilon} = b(\cos\epsilon + i\sin\epsilon) = B + iC$$

The Jones vector for this general case is then

$$\tilde{E}_0 = \begin{bmatrix} A \\ B + iC \end{bmatrix}$$

The ellipse is situated in a rectangle of sides $2E_{0x}$ and $2E_{0y}$. In terms of the parameters A, B, and C, the derivation of Eq. (14-9) makes clear that

$$E_{0x} = A, \qquad E_{0y} = \sqrt{B^2 + C^2}, \qquad \text{and} \quad \epsilon = \tan^{-1}\left(\frac{C}{B}\right) \qquad (14\text{-}11)$$

Example

Analyze the Jones vector given by

$$\begin{bmatrix} 3 \\ 2 + i \end{bmatrix}$$

to show that it represents elliptically polarized light.

the whose Jones vector is by Eq.

E_x leads E_y by some angle ϵ, that is, $\varphi_y - \varphi_x = \epsilon$. Taking $\varphi_x = 0$, $\varphi_y = \epsilon$, $E_{0x} = A$, and $E_{0y} = b$, the Jones vector is

$$\tilde{E}_0 = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}e^{i\varphi_y} \end{bmatrix} = \begin{bmatrix} A \\ be^{i\epsilon} \end{bmatrix}$$

Using Euler's theorem we write

$$be^{i\epsilon} = b(\cos \epsilon + i \sin \epsilon) = B + iC$$

The Jones vector for this general case is then

$$\tilde{E}_0 = \begin{bmatrix} A \\ B + iC \end{bmatrix} \qquad (14\text{-}9)$$

is inclined at an angle a with to the x-axis, as shown in Figure 14-7. The angle of inclination is determined from

the x-axis, as shown in Figure 14-7. The angle of inclination is determined from

$$\tan 2\alpha = \frac{2E_{0x}E_{0y}\cos \epsilon}{E_{0x}^2 - E_{0y}^2} \qquad (14\text{-}10)$$

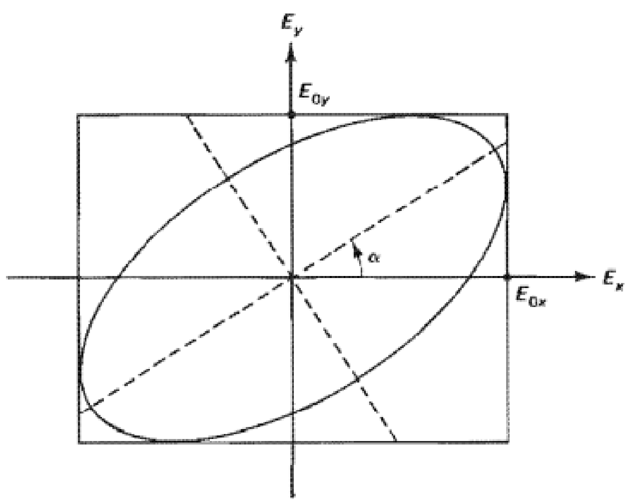


Figure 14-7 Elliptically polarized light oriented at an angle relative to the x-axis.

Example

Consider the result of allowing left-circularly polarized light to pass through an eighth-wave plate.

Solution We first need a matrix that can represent the eighth-wave plate, a phase retarder that introduces a relative phase of $2\pi/8 = \pi/4$, or 45°. Thus, letting $\epsilon_x = 0$,

$$M = \begin{bmatrix} e^{i\epsilon_x} & 0 \\ 0 & e^{i\epsilon_y} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

This matrix is then allowed to operate on the Jones vector representing the left-circularly polarized light:

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} 1 \\ ie^{i\pi/4} \end{bmatrix} = \begin{bmatrix} 1 \\ e^{i3\pi/4} \end{bmatrix}$$

to show that it represents elliptically polarized light.

Solution The light has relative phase between component vibrations of $\varphi_y - \varphi_x = \epsilon = \tan^{-1}(\frac{1}{2}) = 26.6°$. Since $E_{0x} = 3$ and $E_{0y} = \sqrt{2^2 + 1^2} = \sqrt{5}$, the inclination angle of the axis is given by

$$\alpha = \frac{1}{2}\tan^{-1}\frac{(2)(3)(\sqrt{5})\cos(26.6°)}{9 - 5} = 35.8°$$

With this data the ellipse can be sketched as indicated in Figure 14-7. More precisely, the equation of the ellipse is given by

$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\epsilon = \sin^2\epsilon \qquad (14\text{-}12)$$

For this example, the equation of the ellipse is

$$\frac{E_x^2}{9} + \frac{E_y^2}{5} - 0.267E_xE_y = 0.2$$

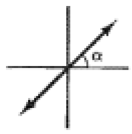
When E_x lags E_y, the phase angle ϵ becomes negative and leads to the Jones vector representing a clockwise rotation instead,

$$E_0 = \begin{bmatrix} A \\ B - iC \end{bmatrix}$$

TABLE 14-1 SUMMARY OF JONES VECTORS $E_0 = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ E_{0y} e^{i\varphi_y} \end{bmatrix}$

I. Linear Polarization ($\Delta\varphi = m\pi$)

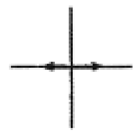
General:



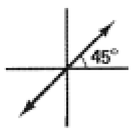
$E_0 = \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$

Vertical: $E_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

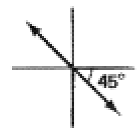
Horizontal: $E_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$



At + 45°: $E_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

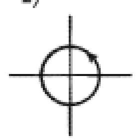


At −45°: $E_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$



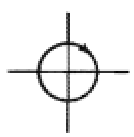
II. Circular Polarization $\left(\Delta\phi = \frac{\pi}{2}\right)$

Left:



$E_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$

Right:

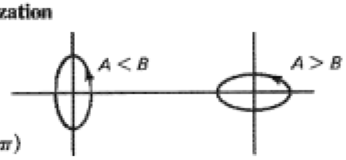


$E_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$

III. Elliptical Polarization

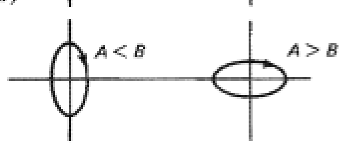
Left:

($\Delta\phi = (m + \frac{1}{2})\,\pi$)



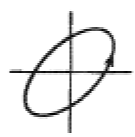
$E_0 = \frac{1}{\sqrt{A^2 + B^2}} \begin{bmatrix} A \\ iB \end{bmatrix}$

Right:



$E_0 = \frac{1}{\sqrt{A^2 + B^2}} \begin{bmatrix} A \\ -iB \end{bmatrix}$

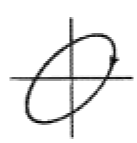
Left:



$E_0 = \frac{1}{\sqrt{A^2 + B^2 + C^2}} \begin{bmatrix} A \\ B + iC \end{bmatrix}$

$\left(\Delta\phi \neq \begin{cases} m\pi \\ (m + \frac{1}{2})\pi \end{cases}\right)$

Right:

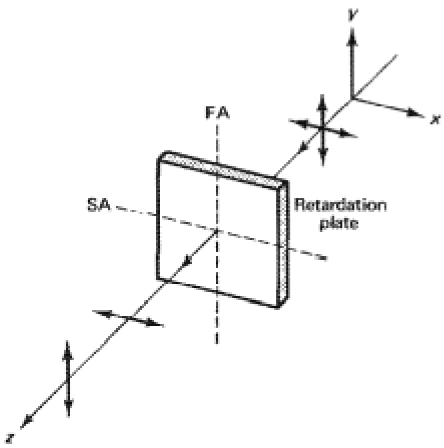


$E_0 = \frac{1}{\sqrt{A^2 + B^2 + C^2}} \begin{bmatrix} A \\ B - iC \end{bmatrix}$

3- Phase Retarder.

The phase retarder does not remove either of the component orthogonal but introduces a phase difference between them. If light to each vibration travels with different speeds through such a retardation plate, there will be a cumulative phase difference $\Delta\varphi$ between the two waves as they emerge.

Figure shows the effect of a retardation plate on unpolarized light in a case where the vertical travels through the plate faster than the horizontal component. This is suggested by the schematic separation of the two components on the optical although of course both waves are simultaneously present at each point along the axis. The fast (FA) and slow axis (SA) directions of the plate are indicated. When the net difference $\Delta\varphi = 90\,°$, the retardation is called a quarter-wave when it is $90\,°$, it is called a half wave plate.



Rotator. The rotator has the effect of rotating the direction of linearly polarized light incident on it by some particular angle. Vertical linearly polarized light is shown incident on a rotator in Figure 14-10. The effect of the rotator element is to transmit linearly polarized light whose direction of vibration has, in this case, rotated counterclockwise by an angle θ.

We desire now to create a set of matrices corresponding to these three types of polarizers so that, just as the optical element alters the polarization mode of the actual light beam, an element matrix operating on a Jones vector will produce the same result mathematically. We adopt a pragmatic point of view in formulating appropriate matrices. For example, consider a linear polarizer with a transmission axis along the vertical, as in Figure 14-8. Let a 2×2 matrix representing the polarizer operate on vertically polarized light, and let the elements of the matrix to be determined

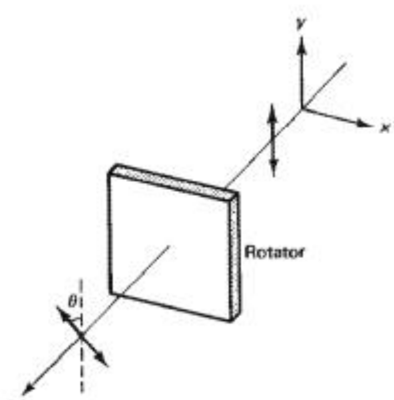


Figure 14-10 Operation of a rotator.

be represented by letters a, b, c, and d. The resultant transmitted or product light in this case must again be vertically linearly polarized light. Symbolically,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

This matrix equation is equivalent to the algebraic equations

$$a(0) + b(1) = 0$$

$$c(0) + d(1) = 1$$

from which we conclude $b = 0$ and $d = 1$. To determine elements a and c, let the same polarizer operate on horizontally polarized light. In this case no light is transmitted, or

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The corresponding algebraic equations are now

$$a(1) + b(0) = 0$$

$$c(1) + d(0) = 0$$

from which $a = 0$ and $c = 0$. We conclude here without further proof that the appropriate matrix is

$$M = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{linear polarizer, TA vertical} \qquad (14\text{-}13)$$

The matrix for a linear polarizer, TA horizontal, can be obtained in a similar manner and is included in Table 14-2, near the end of this chapter. Suppose next that the linear polarizer has a TA inclined at 45° to the x-axis. To keep matters as simple as possible we allow light linearly polarized in the same direction as—and perpendicular to—the TA to pass in turn through the polarizer. Following the approach used earlier,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a + b = 1$$

$$c + d = 1$$

$$a - b = 0$$

$$c - d = 0$$

or $a = b = c = d = \frac{1}{2}$. Thus the correct matrix is

$$M = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \text{linear polarizer, TA at } 45^\circ \tag{14-14}$$

In the same way, a general matrix representing a linear polarizer with TA at angle θ can be determined. This is left as an exercise for the student. The result is

$$M = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix} \tag{14-15}$$

which includes Eqs. (14-13) and (14-14) as special cases, with $\theta = 90^\circ$ and $\theta = 45^\circ$, respectively.

Proceeding to the case of a phase retarder, we desire a matrix that will transform the elements

$$E_{0x}e^{i\varphi_x} \quad \text{into} \quad E_{0x}e^{i(\varphi_x + \epsilon_x)}$$

and

$$E_{0y}e^{i\varphi_y} \quad \text{into} \quad E_{0y}e^{i(\varphi_y + \epsilon_y)}$$

Inspection is sufficient to show that this is accomplished by the matrix operation

$$\begin{bmatrix} e^{i\epsilon_x} & 0 \\ 0 & e^{i\epsilon_y} \end{bmatrix}\begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}e^{i\varphi_y} \end{bmatrix} = \begin{bmatrix} E_{0x}e^{i(\varphi_x+\epsilon_x)} \\ E_{0y}e^{i(\varphi_y + \epsilon_y)} \end{bmatrix}$$

Thus the general form of a matrix representing a phase retarder is

$$M = \begin{bmatrix} e^{i\epsilon_x} & 0 \\ 0 & e^{i\epsilon_y} \end{bmatrix} \qquad \text{phase retarder} \tag{14-16}$$

where ϵ_x and ϵ_y represent the advance in phase of the E_x- and E_y-components of the incident light. Of course, ϵ_x and ϵ_y may be negative quantities. As a special case, consider a quarter-wave plate (QWP) for which $|\Delta\epsilon| = \pi/2$. We distinguish the case for which $\epsilon_y - \epsilon_x = \pi/2$ (SA vertical) from the case for which $\epsilon_x - \epsilon_y = \pi/2$ (SA horizontal). In the former case, then, let $\epsilon_x = -\pi/4$ and $\epsilon_y = +\pi/4$. Obviously, other choices—an infinite number of them—are possible, so that Jones matrices, like Jones vectors, are not unique. This particular choice, however, leads to a common form of the matrix, due to its symmetrical form:

$$M = \begin{bmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} = e^{-i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \text{QWP, SA vertical} \tag{14-17}$$

common form of the matrix, due to its symmetrical form:

$$M = \begin{bmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} = e^{-i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \text{QWP, SA vertical} \qquad (14\text{-}17)$$

Similarly, when $\epsilon_x > \epsilon_y$,

$$M = e^{i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \qquad \text{QWP, SA horizontal} \qquad (14\text{-}18)$$

Corresponding matrices for half-wave plates (HWP), where $|\Delta\epsilon| = \pi$, are given by

$$M = \begin{bmatrix} e^{-i\pi/2} & 0 \\ 0 & e^{i\pi/2} \end{bmatrix} = e^{-i\pi/2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \text{HWP, SA vertical} \qquad (14\text{-}19)$$

$$M = \begin{bmatrix} e^{i\pi/2} & 0 \\ 0 & e^{-i\pi/2} \end{bmatrix} = e^{i\pi/2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \text{HWP, SA horizontal} \qquad (14\text{-}20)$$

The elements of the matrices are identical in this case, since advancement of phase by π is physically equivalent to retardation by π. The only difference lies in the prefactors that modify the phases of all the elements of the Jones vector in the same way and hence do not affect interpretation of the results.

The requirement for a rotator of angle β is that an **E**-vector oscillating linearly at angle θ be converted to one that oscillates linearly at angle $(\theta + \beta)$. Thus the matrix elements must satisfy

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \cos(\theta + \beta) \\ \sin(\theta + \beta) \end{bmatrix}$$

or

$$a\cos\theta + b\sin\theta = \cos(\theta + \beta)$$

$$c\cos\theta + d\sin\theta = \sin(\theta + \beta)$$

From the trigonometric identities for the sine and cosine of the sum of two angles,

$$\cos(\theta + \beta) = \cos\theta\cos\beta - \sin\theta\sin\beta$$

$$\sin(\theta + \beta) = \sin\theta\cos\beta + \cos\theta\sin\beta$$

it follows that

$$a = \cos\beta \qquad c = \sin\beta$$

$$b = -\sin\beta \qquad d = \cos\beta$$

so that the desired matrix is

$$M = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \qquad \text{rotator through angle } +\beta \qquad (14\text{-}21)$$

The Jones matrices derived in this chapter are summarized in Table 14-2. As an important example, consider the production of circularly polarized light by

TABLE 14-2 SUMMARY OF JONES MATRICES

I. Linear polarizers					
TA horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	TA vertical	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	TA at 45° to horizontal	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
II. Phase retarders					
		General	$\begin{bmatrix} e^{i\epsilon_x} & 0 \\ 0 & e^{i\epsilon_y} \end{bmatrix}$		
QWP, SA vertical	$e^{-i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	QWP, SA horizontal	$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$		
HWP, SA vertical	$e^{-i\pi/2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	HWP, SA horizontal	$e^{i\pi/2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$		
III. Rotator					
Rotator	$(\theta \to \theta + \beta)$	$\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$			

Example

Consider the result of allowing left-circularly polarized light to pass through an eighth-wave plate.

Solution We first need a matrix that can represent the eighth-wave plate, a phase retarder that introduces a relative phase of $2\pi/8 = \pi/4$, or 45°. Thus, letting $\epsilon_x = 0$,

$$M = \begin{bmatrix} e^{i\epsilon_x} & 0 \\ 0 & e^{i\epsilon_y} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

This matrix is then allowed to operate on the Jones vector representing the left-circularly polarized light:

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} 1 \\ ie^{i\pi/4} \end{bmatrix} = \begin{bmatrix} 1 \\ e^{i3\pi/4} \end{bmatrix}$$

The resultant Jones vector indicates that the light is elliptically polarized, and the components are out of phase by 135°. Using Euler's equation to expand $e^{i3\pi/4}$, we obtain

$$e^{i3\pi/4} = -\frac{1}{\sqrt{2}} + i\left(\frac{1}{\sqrt{2}}\right)$$

and using our standard notation for this case, we have

$$M = \begin{bmatrix} A \\ B + iC \end{bmatrix}, \quad \text{where } A = 1,\ B = -\frac{1}{\sqrt{2}},\ \text{and } C = \frac{1}{\sqrt{2}}$$

Comparing this matrix with the general form in Eq. (14-5), we determine that $E_{0x} = 1$ and $E_{0y} = 1$. Making use of Eq. (14-10), we also determine that $\alpha = -45°$.