

# Chapter 1

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The errors :-

Algorithm :- A set of rules that precisely defines a sequence of operations to solve a given problem.

Numerical analysis :- A branch of mathematics that studied and concerned with methods and algorithms for obtaining approximating solutions to problems.

Types of errors :-

① Absolute error :- The difference between the true value  $x$  and the approximate value  $\hat{x}$  and denoted by  $\Delta x$ .

$$\Delta x = x - \hat{x}$$

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② Relative error: The ratio of the absolute error  $\Delta x$  to the true value  $x$  and denoted by  $\delta_x$ .

$$\delta_x = \left| \frac{\Delta x}{x} \right|$$

Example: If  $x = 0.008$  and  $\hat{x} = 0.007$ , find the absolute and relative errors?

Solution:  $\Delta x = x - \hat{x} = 0.008 - 0.007 = 0.001$

$$\delta_x = \left| \frac{\Delta x}{x} \right| = \left| \frac{0.001}{0.008} \right| = \left| \frac{1}{8} \right| = 0.125$$

Example: If  $x = 100$  and  $\hat{x} = 97$ , find  $\Delta x$  and  $\delta_x$ ?

Solution:

$$\Delta x = x - \hat{x} = 100 - 97 = 3$$

$$\delta_x = \left| \frac{\Delta x}{x} \right| = \left| \frac{3}{100} \right| = 0.03$$

# Sources of errors :-

## 1- Round off error :-

A computer only represent a number approximately for example a number like  $\frac{1}{3}$  maybe represented as 0.3333333 on a PC.

Then the round off error in this case is

$$\frac{1}{3} - 0.3333333 = 0.000000033$$

also like  $\pi$ ,  $\sqrt{2}$ , ...

## 2- Truncation error :-

Truncation error is defined on the error caused by truncation a mathematical procedure.

For example, the Maclaurin series is given by :-

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

The series has infinite number of terms to calculate  $e^x$ , then  $e^x \approx 1 + x + \frac{x^2}{2!}$

The truncation error such an approximation is

$$e^x - \left[ 1 + x + \frac{x^2}{2!} \right] = \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$