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2- Analytical method

Analytical method based on the mean-value theorem. Let $f(x)$ be a real continuous function on the interval $[a,b]$, where a and b real numbers such that $a<b$, if $f(a)$ and $f(b)$ different in signs then there exists at least one real root on the interval $[a,b]$.

- The accuracy to determine the location of roots depend on the divided of the interval $[a,b]$ into subintervals.

- The number of positive roots of $f(x)$ is the number of change in signs of $f(x)$.
The number of negative roots of $f(x)$ is the number of change in signs of $f(-x)$.

Example:

Find the location of the roots of the following equation by analytic method on the interval $[-1,1]$, $f(x)=x^2-x-1$?

Solution:

$$f(x) = +x^2 - x - 1$$

$\therefore$ there is one +ve root.

$$f(-x) = x^2 + x - 1$$

$\therefore$ there is one -ve root

x	-1	1
$f(x)$	$+$	$-$

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Example: Find the locations of the roots of the following equations by analytical method:-

① $f(x) = x^2 - x - 1$ in $[-2, 2]$?

Solution: f have positive and negative roots

x	-2	-1	0	1	2
$f(x)$	$+$	$+$	$-$	$-$	$+$

∴ there are two roots in the $[-1, 0]$ and $[1, 2]$

② $f(x) = x^3 - 5x^2 + 2x + 8$, in $I = [-4, 4]$?

Solution:

$$f(x) = x^3 - 5x^2 + 2x + 8$$

there are two +ve roots.

$$f(-x) = -x^3 - 5x^2 - 2x + 8$$

there is one −ve root.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	$-$	$-$	$-$	0	$+$	$+$	0	$-$	0

∴ -1, 2 and 4 are the roots.