

Numerical methods:

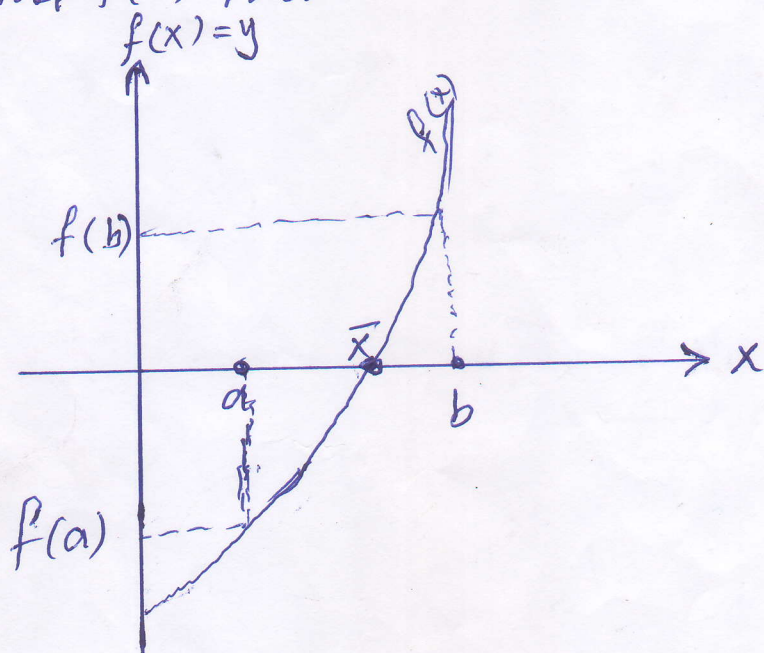
1) Bisection method:-

- Let $f(x)$ be a continuous function of (x) on the interval $[a, b]$.

- The equation $f(x) = 0$ has a root in the interval $[a, b]$ if the following relation holds:

$$f(a) * f(b) < 0$$

i.e. $f(a)$ and $f(b)$ have different signs.



Steps of Algorithm:-

1. Choose an interval $[a, b]$ such that $f(a) * f(b) < 0$

2. Find the value of (x_i) by dividing the distance between a and b :
$$x_i = \frac{a+b}{2}$$

3. Calculate $f(x_i)$ (by using x_i -value).

4. If $f(a) * f(x_i) < 0 \Rightarrow b = x_i$

If $f(a) * f(x_i) > 0 \Rightarrow a = x_i$

5. Repeat the above procedure starting from step 2) to calculate a new (x_i) --- and so on.

6. Terminate the calculations when the given accuracy condition is satisfied.

We can use one of the following stopping

Conditions :-

(a) $|x_{i+1} - x_i| \leq \epsilon$

(d) $\left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| * 100 \leq \epsilon$

(b) $\left| \frac{x_{i+1} - x_i}{x_i} \right| \leq \epsilon$

(c) $|f(x_i)| \leq \epsilon$

Example : Find the approximate root of the following equation by using bisection method :

$3x^2 = x + 5.25$ in the interval $[0.5, 3]$ and

$\epsilon = 0.4$?

1)
Solution

$$a = 0.5, b = 3, \epsilon = 0.4, f(x) = 3x^2 - x - 5.25$$

$$x_i = \frac{a+b}{2}, f(a) * f(x_i) < 0 \Rightarrow b = x_i$$

$$f(a) * f(x_i) > 0 \Rightarrow a = x_i$$

the condition of stopping is $|f(x_i)| < 0.4$

a	b	x_i	$f(a)$	$f(x_i)$	$f(a) * f(x_i)$
0.5	3	1.75	-5	2.1875	-
0.5	1.75	1.125	-5	-2.5781	+
1.125	1.75	1.4375	-2.5781	-0.4882	+
1.4375	1.75	1.5937	-0.4882	0.7759	-
1.4375	1.5937	1.5156	-0.4882	0.1255	

∴ $\bar{x} = 1.5156$ is a root.

Exercise Find the approximate root of the following equation by using bisection method?
 $2x - 1 = 0$ in the interval $[0, 2]$?

(12)

example: Use bisection method to determine the positive root of the equation

$$e^{-x} = x \quad \text{and} \quad \epsilon = 6\% ?$$

Solution:

$$f(x) = e^{-x} - x.$$

To find the interval for (x) :

x	0	1
$f(x)$	1	-0.63

 $\Rightarrow \begin{cases} a=0 \Rightarrow f(a)=1 \\ b=1 \Rightarrow f(b)=-0.63 \end{cases}$

$\therefore a=0$ and $b=1$

We have $f(a) * f(b) < 0 \Rightarrow$ there is a root in the interval $[0, 1]$.

i	a	b	x_i	$\epsilon\%$	$f(a)$	$f(x_i)$	$f(a) * f(x_i)$
1	0	1	0.5	---	+	+	+
2	0.5	1	0.75	$\simeq 33\%$	+	-	-
3	0.5	0.75	0.625	$\simeq 20\%$	+	-	-
4	0.5	0.625	0.5625	$\simeq 11\%$	+	+	+
5	0.5625	0.625	0.5937	$\simeq 5.2\%$	+	-	-

\therefore the root is: $\bar{x} \simeq 0.5937$