Exercise: By N-R method find the solution of the following equation, correct to $|X_{i+1} - X_i| < 0.001$ and use $X_0 = 2$?

4- Fixed point iteration method:-

- Given $f(x) = 0$, write x in terms of $x = g(x)$.
- Label left side as x_{i+1} and right side with x_i.
- Pick x_1 and plug into equation.
- Repeat until converges.

Example 1:

By the fixed point iteration method, find the value of the root of $f(x) = x^2 + 2x - 1$ correct to $|X_{i+1} - X_i| < 0.001$?

Solution

x	0	1
$f(x)$	$-$	$+$

$$X_0 = \frac{0+1}{2} = 0.5$$

(a) $x^2 + 2x - 1 = 0$

$$x^2 = 1 - 2x$$

$$x = \underbrace{\sqrt{1-2x}}_{g(x)}$$

$$g'(x) = \frac{-1}{\sqrt{1-2x}}$$

$$g'(x_0) = \frac{-1}{\sqrt{1-2*0.5}} = \frac{-1}{0} = \infty \quad (\text{divergent})$$

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(b) $2x = 1 - x^2$

$$x = \frac{1-x^2}{2}$$

$$g(x) = \frac{1-x^2}{2}$$

$$g'(x) = -x$$

$$g'(x_0) = -0.5$$

$$|g'(x_0)| = 0.5 < 1 \text{ (convergent}$$

(c) $x^2 + 2x - 1 = 0$

$$x(x+2) - 1 = 0$$

$$x = \frac{1}{x+2} \Rightarrow g(x) = \frac{1}{x+2}$$

$$g'(x) = \frac{-1}{(x+2)^2}$$

$$g'(x_0) = \frac{-1}{(2.5)^2} = \frac{-1}{6.25} = -0.16$$

$$\therefore |g'(x)| = 0.16 < 1 \text{ (Convergent)}$$

Remark: Sometimes we can write $f(x)$ in many formulas of $g(x)$, some of these $g(x)$ convergence to the root and other of $g(x)$ divergence from the root. The following testing defines the best formulas which convergent to the root:

$$|g'(x_0)| < 1$$

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we select the formula:

$$g(x) = \frac{1}{x+2}.$$

$$x_{i+1} = \frac{1}{x_i + 2} \quad , \; x_0 = 0.5$$

i	x_i	x_{i+1}	$\lvert x_{i+1} - x_i \rvert$
0	0.5	0.4	—
1	0.4	0.410	0.010
2	0.410	0.414	0.004
3	0.414	0.414	0

$\therefore$ The root $\bar{x} \cong 0.414$

Exercise:

Find the root of the equation $\sin x - x^2 + 1 = 0$ by fixed point iteration method correct to $\lvert x_{i+1} - x_i \rvert < 0.002$ and $x_0 = 1.5$?