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2- Gauss - Jordan elimination:

In this method, the augmented matrix is converted as shown:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \vdots & c_1 \\ a_{21} & a_{22} & a_{23} & \vdots & c_2 \\ a_{31} & a_{32} & a_{33} & \vdots & c_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & c_1' \\ 0 & 1 & 0 & \vdots & c_2' \\ 0 & 0 & 1 & \vdots & c_3' \end{bmatrix}$$

In this case, we have:

$$1 * X_1 = c_1' \Rightarrow X_1 = c_1'$$
$$1 * X_2 = C_2' \Rightarrow X_2 = C_2'$$
$$1 * X_3 = C_3' \Rightarrow X_3 = C_3'$$

- Elimination procedure:-

* The augmented matrix:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \vdots & c_1 \\ a_{21} & a_{22} & a_{23} & \vdots & c_2 \\ a_{31} & a_{32} & a_{33} & \vdots & c_3 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

* To eliminate a_{21} and a_{31}:

First divide R_1 by a_{11}: New $R_1 = R/a_{11}$

we get

$$\begin{bmatrix} 1 & a_{12}' & a_{13}' & \vdots & c_1' \\ a_{21} & a_{22} & a_{23} & \vdots & c_2 \\ a_{31} & a_{32} & a_{33} & \vdots & c_3 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

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* New $R_2 = R_2 - R_1 * a_{21}$
New $R_3 = R_3 - R_1 * a_{31}$
we get:

$$\begin{array}{c}\left[\begin{array}{ccc:c} 1 & a'_{12} & a'_{13} & c'_1 \\ 0 & a'_{22} & a'_{23} & c'_2 \\ 0 & a'_{32} & a'_{33} & c'_3 \end{array}\right]\end{array}\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

*To eliminate a_{12} and a_{32}
First divide R_2 by a_{22}: New $R_2 = R_2/a_{22}$
we get:

$$\left[\begin{array}{ccc:c} 1 & a'_{12} & a'_{13} & c'_1 \\ 0 & 1 & a''_{23} & c''_2 \\ 0 & a'_{32} & a'_{33} & c'_3 \end{array}\right]\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

* New $R_1 = R_2 - R_2 * a_{12}$
New $R_3 = R_3 - R_2 * a_{32}$

we get:

$$\left[\begin{array}{ccc:c} 1 & 0 & a''_{13} & c''_1 \\ 0 & 1 & a''_{23} & c''_2 \\ 0 & 0 & a''_{33} & c''_3 \end{array}\right]\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

*To eliminate a_{13} and a_{23}:
New $R_3 = R_3/a_{33}$, we get

$$\left[\begin{array}{ccc:c} 1 & 0 & a_{13} & c''_1 \\ 0 & 1 & a_{23} & c''_2 \\ 0 & 0 & 1 & c'''_3 \end{array}\right]\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

* New $R_1 = R_1 - R_3 * a_{13}$

New $R_2 = R_2 - R_3 * a_{23}$

* we get

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & C_1''' \\ 0 & 1 & 0 & C_2''' \\ 0 & 0 & 1 & C_3''' \end{array}\right]$$

* The solution are: $X_1 = C_1'''$, $X_2 = C_2'''$, $X_3 = C_3'''$.

Example Solve the following system of linear equations by using Gauss-Jordan elimination method:

$$3X_1 - 6X_2 + 7X_3 = 3$$
$$9X_1 \qquad - 5X_3 = 3$$
$$5X_1 - 8X_2 + 6X_3 = -4$$

Sol. The augmented matrix is:

$$\begin{array}{c} \left[\begin{array}{ccc|c} 3 & -6 & 7 & 3 \\ 9 & 0 & -5 & 3 \\ 5 & -8 & 6 & -4 \end{array}\right] \begin{array}{l} R_1 = R_1/3 \\ R_2 \\ R_3 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 2.333 & 1 \\ 9 & 0 & -5 & 3 \\ 5 & -8 & 6 & -4 \end{array}\right] \begin{array}{l} R_1 \\ R_2 = R_2 - \\ R_3 = R_3 - \end{array} \end{array}$$

$$\left[\begin{array}{ccc} 1 & -2 & 2.333 \\ 0 & 18 & -25.997 \\ 0 & 2 & -5.665 \end{array}\right] \begin{array}{l} R_1 \\ R_2 = R_2/18 \\ R_3 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 2.333 & 1 \\ 0 & 1 & -1.444 & -0.333 \\ 0 & 2 & -5.665 & -9 \end{array}\right] R_2$$

$R_1 = R_1 - (-2)R_2$, $R_3 = R_3 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -0.555 & 0.334 \\ 0 & 1 & -1.444 & -0.333 \\ 0 & 0 & -2.777 & -8.334 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 = R_3 / -2.777 \end{array}$$

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$$\left[\begin{array}{ccc|c} 1 & 0 & -0.555 & 0.334 \\ 0 & 1 & -1.444 & -0.333 \\ 0 & 0 & 1 & 3.001 \end{array}\right] \begin{array}{l} R_1 = R_1 - (-0.555) R_3 \\ R_2 = R_2 - (1.444) R_3 \\ R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1.999 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3.001 \end{array}\right]$$

$\therefore$ The solution are

$x_1 = 1.999$, $x_2 = 4$, $x_3 = 3.001$

Exercise

Solve the following system of linear equations by using Gauss-Jordan elimination method:

$$x_1 - x_2 + x_3 = -4$$
$$5x_1 - 4x_2 + 3x_3 = -12$$
$$2x_1 + x_2 + x_3 = 11$$