(b) Gauss-Seidel method :-

Let

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

be a system of linear equations.

We can find the values of x_1, x_2 and x_3 by using the following formulas :-

$$x_1^{(k+1)} = \left[b_1 - \left(a_{12}x_2^{(k)} + a_{13}x_3^{(k)}\right)\right] / a_{11}$$

$$x_2^{(k+1)} = \left[b_2 - \left(a_{21}x_1^{(k+1)} + a_{23}x_3^{(k)}\right)\right] / a_{22}$$

$$x_3^{(k+1)} = \left[b_3 - \left(a_{31}x_1^{(k+1)} + a_{32}x_2^{(k+1)}\right)\right] / a_{33}$$

$k = 0, 1, 2, \ldots$

Example:

By Gauss-Seidel method, solve the following system:

$$4x_1 + x_2 - 2x_3 = 1$$
$$x_1 + 3x_2 - x_3 = 8$$
$$x_1 - 7x_2 + 10x_3 = 2$$

where $x_1^{(0)} = 1$, $x_2^{(0)} = 3$, $x_3^{(0)} = 2$, two loops?

(2)

Solution

$$X_1^{(K+1)} = [b_1 - (a_{12} X_2^{(K)} + a_{13} X_3^{(K)})] / a_{11}$$

$$X_2^{(K+1)} = [b_2 - (a_{21} X_1^{(K+1)} + a_{23} X_3^{(K)})] / a_{22}$$

$$X_3^{(K+1)} = [b_3 - (a_{31} X_1^{(K+1)} + a_{32} X_2^{(K+1)})] / a_{33}$$

$$K = 0, 1, 2, \ldots$$

$$X_1^{(1)} = [1 - (X_2^{(0)} - 2 X_3^{(0)})] / 4 = [1-(3-4)] / 4 = 0.5$$

$$X_2^{(1)} = [8 - (X_1^{(1)} - X_3^{(0)})] / 3 = [8-(0.5-2)] / 3 = 3.167$$

$$X_3^{(1)} = [2 - (X_1^{(1)} - 7 X_2^{(1)})] / 10 = [2-(0.5-7(3.167))] / 10$$
$$= 2.367$$

$$X_1^{(2)} = [1 - (X_2^{(1)} - 2 X_3^{(1)})] / 4 = [1-(3.167 - 2(2.367))] / 4$$
$$= 0.643$$

$$X_2^{(2)} = [8 - (X_1^{(2)} - X_3^{(1)})] / 3 = [8-(0.643 - 2.367)] / 3$$
$$= 3.241$$

$$X_3^{(2)} = [2 - (X_1^{(2)} - 7 X_2^{(2)})] / 10$$
$$= [2-(0.643 - 7(3.241))] / 10 = 2.403$$

③

Exercise:-

Solve the following system of linear equations, by Gauss-Seidel method:

$$-3X_1 + X_2 - X_3 = -1$$
$$3X_1 + 9X_2 + X_3 = 2$$
$$X_1 - 2X_2 + 4X_3 = 0$$

If $X_1^{(0)} = X_2^{(0)} = X_3^{(0)} = 0$, three loops?