Functions of two or More varibles:-

Def:- suppose D is a collection of n-tuples of real number $(x_1, x_2, ---, x_n)$

A Function f with domain D is a rule that assigns a number $w = f(x_1, x_2, ----, x_n)$ to each n-tuples in D.

The set of values $w = f(x_1, x_2, ---, x_n)$ is called the rang of f

The symbol w is called dependent varible of f and $x_1, ---, x_n$ are called the independent varibles.

output

input

Ex, $v = r^2 \pi h$

In the Function v, the dependent varible is v and the independent varible are r and h.

Remark:- Function given by Formula are evaluated in the usual way subtituting values of the independing varibles and calculating the corresponding values of the independent varible.

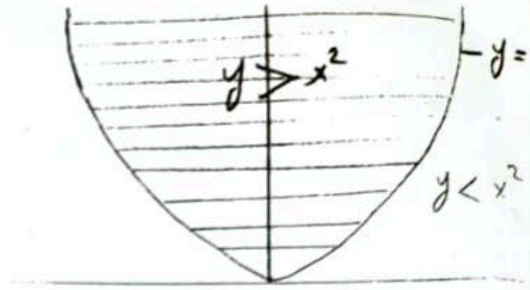
Ex, 1. The value of the Function $w = \sqrt{x^2+y^2+z^2}$ at the point $(3, 0, 4)$ is $\sqrt{9+0+16} = \sqrt{25} = 5$

2. The value of the Function $z = x^2 + y^2$ at $(2, 5)$ is $4 + 25 = 29$.

Ex, Sketch the domain of $f(x,y)=\sqrt{y-x^2}$, what the Function rang ?

sol, $y-x^2 \geq 0 \Rightarrow y \geq x^2$

$D_f = \{(x,y) \in \mathbb{R}^2 : y-x^2 \geq 0\}$
$= \{(x,y) \in \mathbb{R}^2 : y \geq x^2\}$.



$\therefore D_f$ is the set of points that Lie above and on $y=x^2$
R_f is the set of all non-negative numbers $z \geq 0$.
$R_f = \{z \mid 0 \leq z < \infty\}$

Ex, Find the domain and the range of $z=f(x,y)=\dfrac{xy}{x^2-y^2}$

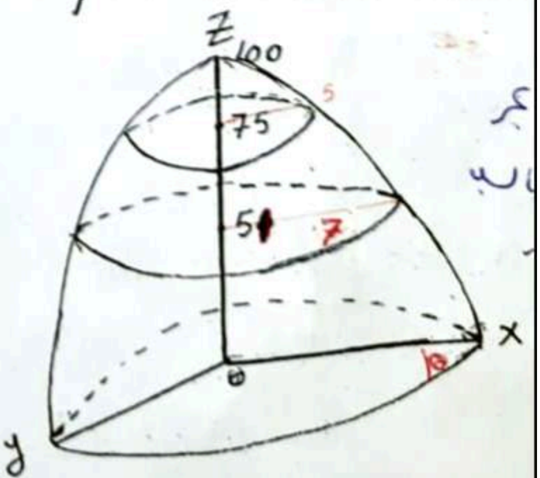
sol, $x^2-y^2 \neq 0 \Rightarrow x^2 \neq y^2 \Rightarrow y \neq \mp x$

D_f = ALL points in the plane except points on the Line $y=x$, y
i.e $D_f = \{(x,y) \in \mathbb{R}^2, y \neq \mp x\}$.

$R_f = \{z : -\infty < z < \infty\} = \{z : z \in \mathbb{R}\} = \mathbb{R}$

Ex, Graph $z=f(x,y)=100-x^2-y^2$ and plot the Level curves $f(x,y)=0, 51, 75$.

sol, The graph is a paraboloid
$D_f = \{(x,y) : (x,y) \in \mathbb{R}^2\}$.
$R_f = \{z : z \leq 100\}$.



1- if $f(x,y)=0$ is the set of points in the xy-plane at whic
$100-x^2-y^2=0 \Rightarrow x^2+y^2=100$ which is the circle in the xy-plan
of radius 10 and center $(0,0,0)$.

2. The curve $f(x,y)=51$ is the set of points in the xy-plane at which $100-x^2-y^2=51 \Rightarrow x^2+y^2=49$ is the circle in the xy-plane of radius 7 and center $(0,0,51)$.

3. The curve $f(x,y)=75$ is the set of points in the xy-plane at which $100-x^2-y^2=75 \Rightarrow x^2+y^2=25$ which is circle in the xy-plane of radius 5 and center $(0,0,75)$.

Limits and Continuity :-

Def:- Let $Z=f(x,y)$ be a function and L be a fixed number is a Limit of $f(x,y)$ as (x,y) approches (x_0,y_0) and we write

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y)=L.$$

Remark :- This definition Like the Limit of Function of one Vorible except that there are two independent varibles involved instead of one.

Theorem ①

1. $\lim\limits_{(x,y)\to(x_0,y_0)} x = x_0.$

2. $\lim\limits_{(x,y)\to(x_0,y_0)} y = y_0$

3. $\lim\limits_{(x,y)\to(x_0,y_0)} K = K.$ .., K is any constant ثابت

Theorem ② IF $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y) = L_1$, $\lim\limits_{(x,y)\to(x_0,y_0)} g(x,y) = L_2$ then

1. $\lim\limits_{(x,y)\to(x_0,y_0)} [f(x,y) \mp g(x,y)] = L_1 \mp L_2$

2. $\lim\limits_{(x,y)\to(x_0,y_0)} [f(x,y) \cdot g(x,y)] = L_1 \cdot L_2$

3. $\lim\limits_{(x,y)\to(x_0,y_0)} [K \cdot f(x,y)] = K \cdot L_1$ (K is any constant).

4. $\lim\limits_{(x,y)\to(x_0,y_0)} \dfrac{f(x,y)}{g(x,y)} = \dfrac{\lim f(x,y)}{\lim g(x,y)} = \dfrac{L_1}{L_2}$ if $L_2 \neq 0$.

Examples:- Find the Limits of the Following Functions:-

1. $\lim\limits_{(x,y)\to(1,1)} (x^2+y^2) = 2.$

2. $\lim\limits_{(x,y)\to(0,1)} \dfrac{x^2-xy+10}{x^2y+2xy+y^2} = \dfrac{10}{1} = 10.$

3. $$\lim_{(x,y)\to(0,0)} \frac{x^2 \sin x}{\sqrt{x^2+3}} = 0$$

$\sin 0 = 0$
$\cos 0 = 1$

4. $$\lim_{(x,y,z)\to(2,4,1)} z = 1.$$

5. $$\lim_{(x,y)\to(0,\ln(2))} e^{x+y} = e^{\ln(2)} = 2$$

6. $$\lim_{(x,y)\to(0,4)} \frac{x}{\sqrt{y}} = \frac{0}{2} = 0$$

7. $$\lim_{(x,y)\to(0,0)} \cos\left(\frac{x^2+y^2}{x+y+1}\right) = \cos(0) = 1$$

8. $$\lim_{(x,y,z)\to(1,1,1)} \sqrt{x^2+y^2+z^2-1} = \sqrt{2}$$

9. $$\lim_{(x,y)\to(1,1)} \cos\sqrt[3]{|xy|-1} = \cos(0) = 1$$