Ex/ prove that the Function $f(x,y) = \begin{cases} \dfrac{x^4-y^4}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ is Continuous at every point.

Sol/ ① Since the Function $f(x,y) = \dfrac{x^4-y^4}{x^2+y^2} = \dfrac{(x^2-y^2)\cancel{(x^2+y^2)}}{\cancel{(x^2+y^2)}}$

$= x^2-y^2$ defined at any point $(x,y) \neq (0,0)$

then the function f is continuous at any point $(x,y) \neq (0,0)$ because it is polynomail function of x and y.

f is defined at $(0,0)$ and $f(0,0) = 0$

② $\lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{(x,y)\to(0,0)} x^2-y^2 = 0$

③ $f(0,0) = \lim\limits_{(x,y)\to(0,0)} f(x,y) = 0$

$\therefore$ f is cont. at every point.

Ex/ Let $f(x,y) = \begin{cases} x^3+4y & (x,y) \neq (1,1) \\ 0 & (x,y) = (1,1) \end{cases}$ Is f cont. at $(1,1)$?

Sol/ 1. $f(1,1) = 0 \Rightarrow f$ is defined at $(1,1)$

2. $\lim\limits_{(x,y)\to(1,1)} f(x,y) = \lim\limits_{(x,y)\to(1,1)} x^3+4y = 5.$

3. $f(1,1) \neq \lim\limits_{(x,y)\to(1,1)} f(x,y)$

$\therefore$ f is not continuous.

Exe. Find the Limit of the Following Functions.

1. $\lim\limits_{(x,y,z)\to(0,1,1)} \dfrac{e^{x+y+z}}{z+\sin\sqrt{x^2y}}$

2. $\lim\limits_{(x,y,z)\to(1,2,2)} \dfrac{\ln\sqrt{x^2+2y^2+z^2}}{xyz}$

Example: show that the function

$$f(x,y)=\begin{cases}\dfrac{x^2-y^2}{x+y}, & (x,y)\text{ not on the line } y=-x\\ 6, & (x,y)\text{ on the line } y=-x\end{cases}$$

is continuous at the point $(3,-3)$

Partial Derivative

Let $z = f(x,y)$ be a Function of two independent varibles x and y

1. IF y is Fixed, then f will be a Function of one varible, then we can derive with respect to (w.r.t) x, this derivative is called partial derivative of f w.r.t x and denoted by f_x or $\frac{\partial f}{\partial x}$, hence f_x is a Function and its value at (x_0, y_0) =

$$\frac{\partial f}{\partial x}(x_0, y_0) \text{ or } f_x(x_0, y_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

2. IF x is fixed, then will be a Function of one varible y, then we derive w.r.t y, this derivative is called partial derivative of f w.r.t y and denoted by f_y or $\frac{\partial f}{\partial y}$ hence f_y is a Function and its value at (x_0, y_0) =

$$\frac{\partial f}{\partial y}(x_0, y_0) \text{ or } f_y(x_0, y_0) = \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

Ex Find f_x, f_y of the Following Functions:-

1. $f(x,y) = 5xy - 7x^2 - y^2 - 3x - 6y - 2$

$f_x = 5y - 14x - 3$

$f_y = 5x - 2y - 6$

2. $f(x,y) = \sqrt{9-x^2-y^2}$

$$f_x = \frac{1}{2}(9-x^2-y^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{9-x^2-y^2}}$$

$$f_y = \frac{-y}{\sqrt{9-x^2-y^2}}$$

H.W Find $f_x(1,2)$, $f_y(1,2)$ for above examples (1) and (2)?

Functions with more than two varibles:-

The definition of the partial derivatives of functions of more than two independent varibles are like the definition for functions of two varibles. They are ordinary derivatives w.r.t one varible, tak while all the other independent varibles as regarded as constant thus if $w = f(x, y, z, u, \omega)$, we have as many as five partial derivatives

IF there are three independent varibles $\boxed{w = f(x,y,z).}$

$$w_x \Big|_{(x_0,y_0,z_0)} = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x, y_0, z_0) - f(x_0,y_0,z_0)}{\Delta x}$$

$$w_y \Big|_{(x_0,y_0,z_0)} = \lim_{\Delta y \to 0} \frac{f(x_0, y_0+\Delta y, z_0) - f(x_0,y_0,z_0)}{\Delta y}$$

$$w_z \Big|_{(x_0,y_0,z_0)} = \lim_{\Delta z \to 0} \frac{f(x_0, y_0, z_0+\Delta z) - f(x_0,y_0,z_0)}{\Delta z}$$

. Find f_x, f_y, f_z in each of the Following problems :-

1. $f(x,y,z) = e^{xyz}$

$f_x = e^{xyz} \cdot (yz)$.

$f_y = e^{xyz} (xz)$.

$f_z = e^{xyz} (xy)$.

2. $f(x,y,z) = \dfrac{x+y+z}{xy+yz+zx}$ (Exc)

3. $f(x,y,z) = x^2 e^{yz} + \cos z^2 + \tan \dfrac{x}{z}$ (Exc).

Def :- 1. The slope of the curve $z = f(x, y_0)$ at $x = x_0$ is defined to be the value of the partial derivative w.r.t x at (x_0, y_0).

2. The tangent to the curve $z = f(x, y_0)$ at $x = x_0$ is defined to be the Line in the plane $y = y_0$ thas pass through the point $((x_0, y_0), f(x_0, y_0))$ with a slope equal to the partial derivative of $f(x_0, y_0)$ w.r.t x at (x_0, y_0).

Ex, Find the tangent to the curve $z = f(x,y) = 10 - xy$ in the plane

1. $y = 4$ at the point $(2, 4, 2)$.

sol, $f_x = -y$, $f_x(2,4) = -4$

$\therefore$ The tangent to the curve f in the plane $y = 4$ is

$$\frac{z - z_0}{x - x_0} = \frac{z-2}{x-2} = -4 \text{ and } y = 4$$

$$z-2=-4x+8 \quad \text{and} \quad y=4$$

$$4x+z=10 \quad \text{and} \quad y=4$$

2. $x=2$ at $(2,4,-2)$.

sol, $f_y=-x \Rightarrow f_y(2,4)=-2$

$\therefore$ The tangent to the curve f in the plane $x=2$ is

$$\frac{z-z_0}{y-y_0}=\frac{z-2}{y-4}=-2 \quad \text{and} \quad x=2.$$

we get $z-2=-2y+8$ and $x=2$

$$z+2y=10 \quad \text{and} \quad x=2$$