Exc

1. Find $\frac{du}{dt}$ if $u = 3xy + e^{xy^2}$, $x = \tan 4t^2$, $y = \sin t \cos^{-1} t$.

2. Find $\frac{dw}{dt}$ if $w = x^4 + 2xy - 5y^2$, $x = \cosh t$, $y = \tan^{-1} t + e^t$

3. Find $\frac{dz}{d\theta}$ if $z = \sqrt{x^2 + y^2}$, $x = \sin^2 \theta$, $y = \ln(\cos\theta)$.

4. if $w = \sqrt{x^2 + 2xy + zx - yz^2}$, $x = \cos^{-1}(r+s)$, $y = \ln(s)$, $z = e^{r^2+s}$

Find $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial s}$.

5. Find $\frac{\partial z}{\partial u}$ if $u=0$, $v=1$, $z = \sin xy + x \sin y$, $x = u^2 v^2$

$y = \frac{u+v}{v^2}$

6. Find $\frac{\partial w}{\partial x}$ at the point $(x,y,z) = (1,1,1)$ If $w = \cos uv$

, $u = xyz$, $v = a(x^2+y^2)$.

7. Find $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial s}$

1. IF $w = \sqrt{\sin^{-1}(x^2+y^2+z^2)}$, $x = \operatorname{sech}^{-1}(r^2+s^2)$, $y = \ln\sqrt{1+s^2}$

$z = \csc\left(\frac{1}{rs}\right)$.

2. IF $w = \operatorname{sech} \frac{xy}{x^2+y^2+z^2} + \log_4\left(\frac{x}{y}\right)$, $x = r^{10} + \sec^{-1}(r^2+s^2+1)$

$y = \operatorname{csch}^{-1}(r^2 + 5s + \sqrt{10})$, $z = \cot^{-1}\left(\frac{1}{rs+r}\right) + 10^r$

Exercises :-

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① By using the chain rule, find $\frac{du}{dt}$ if

$u = 3xy + e^{xy^2}$, $x = \tan 4t^2$, $y = \sin t \cos^{-1} t$, $-1<t<1$

solution :-

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}$$

$$= (3y + y^2 e^{xy^2}) \cdot ((\sec^2 4t^2) \cdot 8t) +$$
$$(3x + 2xy e^{xy^2})(\sin t \, (-\frac{1}{\sqrt{1-t^2}}) + \cos^{-1} t \cdot \cos t)$$

$$= (3 \sin t \cos^{-1} t + (\sin t \cos^{-1} t)^2 e^{\tan 4t^2 (\sin t \cos^{-1} t)^2})$$
$$\cdot 8t \sec^2 4t^2 + (3 \tan 4t^2 + 2 \tan 4t^2 \sin t \cos^{-1} t$$
$$\cdot e^{\tan 4t^2 (\sin t \cos^{-1} t)^2} \cdot (\frac{-\sin t}{\sqrt{1-t^2}} + \cos t \cos^{-1} t)$$

② By using the chain rule, find $\frac{dw}{dt}$ if

$w = x^4 + 2xy - 5y^2$, $x = \cosh t$, $y = \tan^{-1} t + e^t$

solution:

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt}$$

$$= (4x^3 + 2y) \sinh t + (2x - 10y)(\frac{1}{1+t^2} + e^t)$$

$$= (4 \cosh^3 t + 2 \tan^{-1} t + 2e^t) \cdot \sinh t$$
$$+ (2 \cosh t - 10 \tan^{-1} t - 10 e^t)(\frac{1}{1+t^2} + e^t)$$

③ By using the chain rules find $\frac{dz}{d\theta}$ if
$z=\sqrt{x^2+y^2}$, $x=\sin^2\theta$, $y=\ln(\cos\theta)$, where $\cos\theta>0$.
solution:-

$$\frac{dz}{d\theta}=\frac{\partial z}{\partial x}\cdot\frac{dx}{d\theta}+\frac{\partial z}{\partial y}\cdot\frac{dy}{d\theta}$$

$$=\frac{2x}{2\sqrt{x^2+y^2}}\cdot 2\sin\theta\cos\theta+\frac{2y}{2\sqrt{x^2+y^2}}\cdot\frac{-\sin\theta}{\cos\theta}$$

$$=\frac{\sin^2\theta\cdot 2\sin\theta\cdot\cos\theta}{\sqrt{\sin^4\theta+(\ln(\cos\theta))^2}}-\frac{\ln(\cos\theta)\cdot\tan\theta}{\sqrt{\cos^4\theta+(\ln(\cos\theta))^2}}$$

$$=\frac{2\sin^3\theta\cdot\cos\theta-\tan\theta\cdot\ln(\cos\theta)}{\sqrt{\sin^4\theta+(\ln(\cos\theta))^2}}$$

④ Find $\frac{\partial z}{\partial u}$, when $u=0$ and $v=1$, if $z=\sin xy+x\sin y$,
$x=u^2v^2$, $y=\frac{u+v}{u^2}$
solution:-

$$\frac{\partial z}{\partial u}=\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$=(y\cos xy+\sin y)\cdot 2uv^2+(x\cos xy+x\cos y)\cdot\frac{1}{v^2}$$

$$=\frac{u+v}{v^2}\cdot\cos(u^3+u^2v)+\sin\left(\frac{u+v}{v^2}\right)\cdot 2uv^2$$

$$+\left(\frac{u+v}{v^2}\cdot\cos(u^3+u^2v)+u^2v^2\cos\left(\frac{u+v}{v^2}\right)\right)\cdot\frac{1}{v^2}$$

Then $\frac{\partial z}{\partial u}=(1\cos 0+\sin 1)0+0=0$ when
$u=0$, $v=1$

5) Find $\frac{\partial w}{\partial x}$ at the point $(x, y, z) = (1, 1, 1)$ if
$w = \cos uv$, $u = xyz$, $v = a(x^2 + y^2)$

Solution

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$= -(v \sin uv) yz + (-u \sin uv) 2ax$$
$$= -a(x^2+y^2)(\sin(ax^3yz + axy^3z)) \cdot yz$$
$$-2ax^2yz(\sin(ax^3yz + axy^3z))$$

then

$$\frac{\partial w}{\partial x} = -a(1+1)(\sin 2a) \cdot 1 - 2a(\sin 2a)$$
$$= -4a \sin 2a$$, at the point $(x, y, z) = (1, 1, 1)$

Exercises :-

1- Find $\frac{dw}{dt}$ if $w = xy + z$, $x = \cos 2t$, $y = \sin 3t$,
$z = 5t$. what is the derivative's value at $t = 0$?

2- Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ if $w = xy + yz + xz$,
$x = u + v$, $y = u - v$, $z = uv$, and then find $\frac{\partial w}{\partial u}$ and
$\frac{\partial w}{\partial v}$ at the point $(u, v) = (\frac{1}{2}, 1)$

3- Find $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial u}{\partial z}$ at the point $(x, y, z) = (\sqrt{3}, 2, 1)$

if $u = \frac{p - q}{q - r}$, $p = x + y + z$, $q = x - y + z$, $r = x + y - z$

Ex Find $\frac{\partial w}{\partial x}$ if $w = x^2+y^2+z^2$ and $z = x^2+y^2$. ① if x and y are independent variables ② if x and z are independent variables.

sol ① with x and y are independent varibles with $z = x^2+y^2$

we have $w = x^2+y^2+(x^2+y^2)^2$

$$\therefore \frac{\partial w}{\partial x} = 2x + 2(x^2+y^2)\cdot 2x = 2x + 4x^3 + 4xy^2.$$

② with x and z are independent waribles with $y^2 = z - x^2$

$$w = x^2 + (z - x^2) + z^2 = z + z^2$$

$$\therefore \frac{\partial w}{\partial x} = 0 \quad , \quad \frac{\partial w}{\partial z} = 1 + 2z.$$

Remark :-

1. $\left(\frac{\partial w}{\partial x}\right)_y$ means $\frac{\partial w}{\partial x}$ with x and y indep.

2. $\left(\frac{\partial w}{\partial x}\right)_z$ means $\frac{\partial w}{\partial x}$ with x and z indep.

3. $\left(\frac{\partial w}{\partial y}\right)_{x,t}$ means $\frac{\partial w}{\partial y}$ with x, y and t indep.

Exs If $w = x^2 + y - z + \sin t$, $x + y = t$

Find 1. $\left(\frac{\partial w}{\partial x}\right)_{y,z}$ 2. $\left(\frac{\partial w}{\partial x}\right)_{t,z}$ 3. $\left(\frac{\partial w}{\partial y}\right)_{z,t}$ 2

$2x-1$

4. $\left(\frac{\partial w}{\partial z}\right)_{x,t}$ 5. $\left(\frac{\partial w}{\partial t}\right)_{x,z}$ 6. $\left(\frac{\partial w}{\partial t}\right)_{z,y}$

1. with x, y, z are indep. $\Rightarrow w = x^2 + y - z + \sin(x+y)$

$\therefore \left(\frac{\partial w}{\partial x}\right)_{y,z} = 2x + \cos(x+y).$