2 inverse method To Solve Linear systems:

$$A \cdot X = B \Longrightarrow A^{-1}(A \cdot X) = A^{-1} \cdot B$$

$$\underbrace{A^{-1} \cdot A}_{= I} \cdot X = A^{-1} \cdot B \Longrightarrow \boxed{X = A^{-1} \cdot B}$$

$$[A \mid I] \Longrightarrow [I \mid A^{-1}]$$

note: $A \cdot A^{-1} = A^{-1} \cdot A = I$

exe. $\begin{array}{l} 5x_1 + 6x_2 = 7 \\ 3x_1 + 4x_2 = 5 \end{array} \Longleftrightarrow \underset{A}{\begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 7 \\ 5 \end{bmatrix}}$

$$[A \mid I] = \left[\begin{array}{cc|cc} 5 & 6 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{5} \times R_1} \left[\begin{array}{cc|cc} 1 & \frac{6}{5} & 1/5 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \times (-3) + R_2} \left[\begin{array}{cc|cc} 1 & 6/5 & 1/5 & 0 \\ 0 & 2/5 & -3/5 & 1 \end{array}\right] \xrightarrow{R_2 \times \frac{5}{2}}$$

$$\left[\begin{array}{cc|cc} 1 & 6/5 & 1/5 & 0 \\ 0 & 1 & -3/2 & 5/2 \end{array}\right] \xrightarrow{R_2 \times (-6/5) + R_1} \left[\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 1 & -3/2 & 5/2 \end{array}\right]$$

$$\underbrace{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}}_{I} \quad \underbrace{\begin{matrix} 2 & -3 \\ -3/2 & 5/2 \end{matrix}}_{A^{-1}}$$

$$A \cdot A^{-1} \overset{?}{=} I$$

$$X = A^{-1} \cdot B = \begin{bmatrix} 2 & -3 \\ -3/2 & 5/2 \end{bmatrix} \begin{bmatrix} 7 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\therefore \boxed{x_1 = 1}, \boxed{x_2 = 2}$$

ex: solve the system by use Inverse method? (46)

$$\left.\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}\right\} \Longleftrightarrow \overset{A}{\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}} \overset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \overset{B}{\begin{bmatrix} 9 \\ 1 \\ 0 \end{bmatrix}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 2 & 4 & -3 & 0 & 1 & 0 \\ 3 & 6 & -5 & 0 & 0 & 1 \end{array}\right] \xrightarrow[(R_1 \times -3) + R_3]{(R_1 \times -2) + R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & \boxed{2} & -7 & -2 & 1 & 0 \\ 0 & 3 & -11 & -3 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{2} \times R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -7/2 & -1 & 1/2 & 0 \\ 0 & 3 & -11 & -3 & 0 & 1 \end{array}\right] \xrightarrow[(R_2 \times -3) + R_3]{(R_2 \times -1) + R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 11/2 & 2 & -1/2 & 0 \\ 0 & 1 & -7/2 & -1 & 1/2 & 0 \\ 0 & 0 & -1/2 & 0 & -3/2 & 1 \end{array}\right] \xrightarrow{R_3 \times -2} \left[\begin{array}{ccc|ccc} 1 & 0 & 11/2 & 2 & -1/2 & 0 \\ 0 & 1 & -7/2 & -1 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 3 & -2 \end{array}\right]$$

$$\xrightarrow[(R_3 \times 7/2) + R_2]{(R_3 \times -11/2) + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -17 & 11 \\ 0 & 1 & 0 & -1 & 11 & -7 \\ 0 & 0 & 1 & 0 & 3 & -2 \end{array}\right] \therefore A^{-1} = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & 2 \end{bmatrix}$$

$$A \cdot A^{-1} = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = ? \text{ check}$$

Now $$X = A^{-1} \cdot B = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 18 - 17 + 0 \\ -9 + 11 + 0 \\ 0 + 3 + 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Longrightarrow \boxed{x = 1 \quad , y = 2 \quad , z = 3}$$

3/ Grammer Rule

Def. consider The system of n Linear equations in n variables;

$A \cdot X = B$ and $|A| \neq 0$; Then $x_j = \frac{|A_j|}{|A|}$

A_j : is a matrix A when repleace column j by matrix B.

A// let $A = \begin{Bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Bmatrix}$, Then $|A| = (a_{11} \cdot a_{22}) - (a_{21} \cdot a_{22})$

ex: Let $A = \begin{Bmatrix} 3 & 5 \\ 3 & 4 \end{Bmatrix}$, Then find $|A|$?

$|A| = (3 \times 4) - (3 \times 5) = 12 - 15 = -3$

B// let $A = \begin{Bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{Bmatrix}$, Then $|A| = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

$+ a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

ex: ① let $A = \begin{Bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 3 & 2 \end{Bmatrix}$, Then find $|A|$? Exc.

② Let $A = \begin{Bmatrix} 1 & 2 & 4 \\ 8 & -6 & 2 \\ 4 & 0 & -3 \end{Bmatrix}$, Then find $|A|$?

$|A| = 1 \begin{vmatrix} -6 & 2 \\ 0 & -3 \end{vmatrix} - 2 \begin{vmatrix} 8 & 2 \\ 4 & -3 \end{vmatrix} + 4 \begin{vmatrix} 8 & -6 \\ 4 & 0 \end{vmatrix}$

$= 18 + 64 + 96 = 178$

exe: solve The by use Grammer Rule?

$$\begin{aligned} x + 2y + 2z &= -1 \\ x + 3y + z &= 4 \\ x + 3y + 2z &= 3 \end{aligned} \iff \underbrace{\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 3 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}}_{B}$$

$$|A| = 1\begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} = 3 - 2 \quad 0 = 1 \neq 0$$

$$|A_1| = \begin{vmatrix} -1 & 2 & 2 \\ 4 & 3 & 1 \\ 3 & 3 & 2 \end{vmatrix} = -1\begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} - 2\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} + 2\begin{vmatrix} 4 & 3 \\ 3 & 3 \end{vmatrix}$$

$$= -7$$

$$|A_2| = \begin{vmatrix} 1 & -1 & 2 \\ 1 & 4 & 1 \\ 1 & 3 & 2 \end{vmatrix} = -1\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix}$$

$$= 4$$

$$|A_3| = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 3 & 4 \\ 1 & 3 & 3 \end{vmatrix} = 1\begin{vmatrix} 3 & 4 \\ 3 & 3 \end{vmatrix} - 2\begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix}$$

$$= -1$$

$$x = \frac{|A_1|}{|A|} = \frac{-7}{1} = -7 \quad , \quad y = \frac{|A_2|}{|A|} = \frac{4}{1} = 4$$

$$z = \frac{|A_3|}{|A|} = \frac{-1}{1} = -1$$