Tangent plane and normal line to a level surface $f(x,y,z)$

IF $f(x,y,z)$ has continuous partial derivatives at $p_0(x_0,y_0,z_0)$ on the level surface $f(x,y,z)=c$, c is constant.

1) The tangent plane to the surface at the point p_0 is

$$(x-x_0)f_x\Big|_{p_0}+(y-y_0)f_y\Big|_{p_0}+(z-z_0)f_z\Big|_{p_0}=0$$

معادلة المستوي المماس للسطح S

يسمى المخطط الديكارتي للدالة $f(x,y,z)=c$ بالسطح ويقال للمستوي المار بالنقطة P_0 الواقعة على سطح $f(x,y,z)$ بالمستوي المماس للسطح في النقطة P_0 اذا كان هذا المستوي عمودياً على $\vec{\nabla f}$ في P_0.

2) The normal line to $S: f(x,y,z)=c$ at P_0 is the line perpendicular to the tangent plane and parallel to $\vec{\nabla f}$ at P_0 given by the eq:

$$x=x_0+t f_x\Big|_{p_0}$$
$$y=y_0+t f_y\Big|_{p_0}$$
$$z=z_0+t f_z\Big|_{p_0}$$

معادلة المستقيم العمودي على السطح $f(x,y,z)=c$

IF $t=0 \Rightarrow x=x_0,\ y=y_0,\ z=z_0$.

IF non of the partial derivative of f is zero at P_0, then the normal line is also given by the eq:

$$\frac{x-x_0}{f_x\big|_{p_0}}=\frac{y-y_0}{f_y\big|_{p_0}}=\frac{z-z_0}{f_z\big|_{p_0}}$$

Ex_2 :- Find the tangent plane to the sphere $x^2+y^2+z^2=4$ at The point $(-1,1,\sqrt{2})$.

Sol/ $f_x = 2x \Rightarrow f_x|_{p_0} = -2$

$f_y = 2y \Rightarrow f_y|_{p_0} = 2$

$f_z = 2z \Rightarrow f_z|_{p_0} = 2\sqrt{2}$

$\therefore$ The tangent plane is $(x-x_0)f_x|_{p_0} + (y-y_0)f_y|_{p_0} + (z-z_0)f_z|_{p_0} = 0$

$\Rightarrow -2(x+1) + 2(y-1) + 2\sqrt{2}(z-\sqrt{2}) = 0$

$\Rightarrow -2x - 2 + 2y - 2 + 2\sqrt{2}\,z - 4 = 0$

$\Rightarrow -2x + 2y + 2\sqrt{2}\,z = 8 \Rightarrow y + \sqrt{2}\,z - x = 4$

$\therefore$ The point $(-1,1,\sqrt{2})$ Satisfy this eq.

$\boxed{Ex_3}$: Find the tangent plane and the normal Line to the surface $x^2+xyz-z^3=1$ at $P_0(1,1,1)$

Sol/ $f_x = 2x+yz \Rightarrow f_x|_{p_0} = 3$

$f_y = xz \Rightarrow f_y|_{p_0} = 1$

$f_z = xy - 3z^2 \Rightarrow f_z|_{p_0} = -2$

Then the Tangent plane to the surface at p_0 is

$3(x-1) + (y-1) - 2(z-1) = 0 \Rightarrow 3x + y - 2z = 2$

The normal line is

$$\frac{x-1}{3} = \frac{y-1}{1} = \frac{z-1}{-2}$$

Tangent plane and Normal Line For a surface $z=f(x,$

The tangent plane to the surface $z=f(x,y)$ at a point $P_0(x_0$
on the surface is :

$$(x-x_0)\,f_x\Big|_{P_0} + (y-y_0)\,f_y\Big|_{P_0} - (z-z_0) = 0$$

The normal Line to the surface at P_0 is

$$x = x_0 + t\,f_x\Big|_{P_0}$$

$$y = y_0 + t\,f_y\Big|_{P_0}$$

$$z = z_0 - t$$

IF non of the partial derivative of f is zero at P_0, then the normal Line is also given by the eq:-

$$\frac{x-x_0}{f_x\Big|_{P_0}} = \frac{y-y_0}{f_y\Big|_{P_0}} = \frac{z-z_0}{-1}$$

Find eq. of the tangent plane and the normal Line to the surface $z = f(x,y) = 8 - x^2 - y^2$ at the point $(1,1,6)$.

Sol/ $f_x = -2x \rightarrow f_x\Big|_{(1,1,4)} = -2$, $f_y = -2y \Rightarrow f_y\Big|_{(1,1,4)} = -2$

The tangent plane is $(x - x_0) f_x\Big|_{P_0} + (y - y_0) f_y\Big|_{P_0} - (z - z_0) = 0$

$$\Rightarrow -2(x-1) - 2(y-1) - (z-6) = 0$$

$$-2x + 2 - 2y + 2 - z + 6 = 0$$

$$-2x - 2y - z + 10 = 0$$

$$2x + 2y + z = -10$$

The normal Line : $x = 1 - 2t$, $y = 1 - 2t$, $z = 6 - t$.

or $\dfrac{x-1}{-2} = \dfrac{y-1}{-2} = \dfrac{z-6}{-1}$