CHAPTER TWO

Solving of non-linear equations (Root finding)

The roots of an equation $f(x)=0$ are the values of the variable x which satisfies the equation $f(x)=0$, i.e. the roots are the values of x which makes $f(x)$ equal to zero.

We shall denote a root of $f(x)=0$ by $\bar{x}$.

For example if we have the equation $xe^x-4=\ln x$, then $xe^x-\ln x-4=0$ where $f(x)=xe^x-\ln x-4$, and we should find the roots of $f(x)=0$.

We shall take some iterative numerical methods for root finding.

In many of these methods we need to find the interval which contains the roots, and remember that the root may be positive or negative.

Example: Find the intervals contains the roots of the equation $x^2+x=5$.

Solution: $x^2+x=5 \Rightarrow x^2+x-5=0$, where $f(x)=x^2+x-5$.

To find a positive root of $f(x)=0$, we make the following table:

x	0	1	2	3
$f(x)$	$-$	$-$	$+$	$+$

which means that there is a positive root in the interval $(1, 2)$.

To find a negative root of $f(x)=0$, we make the following table:

x	0	-1	-2	-3	-4
$f(x)$	$-$	$-$	$-$	$+$	$+$

which means that there is a negative root in the interval $(-3, -2)$

Remark: Each approximate value to a root is called an لحظي instantaneous root. The instantaneous roots will be denoted by $x_1, x_2, x_3, \dots, x_i, \dots$ or $x_0, x_1, x_2, \dots, x_i, \dots$. When the instantaneous root x_i becomes closer to the real root $\bar{x}$, the function $f(x_i)$ becomes closer to zero, i.e. if $x_i \longrightarrow \bar{x}$, then $f(x_i) \longrightarrow 0$.

In general, full accuracy of finding the roots is نادراً rarely obtained in numerical methods, and we may consider the value of x_i which makes $|f(x_i)| \leq \epsilon$ as a root where ϵ

is a small value, and the smaller ϵ we have the high accuracy we get.

Accuracy may be checked by any of the following ways:

1) The absolute error $e_a = E_{abs} = |x_{i+1} - x_i| \leq \epsilon$.

2) The relative error $e_r = E_{rel} = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \leq \epsilon$

3) The percentage error $e\% = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \times 100 \leq \epsilon$.

تطابق

4) Coincide in decimal digits for the values of x_i and x_{i+1} correct to 2D (2 decimals) or correct to 3D (3 decimals), ... for some i.

There are many methods for solving non-linear equations which are:

1) Graphical method
2) Analytical method
3) Bisection method
4) Newton-Raphson method
5) False Position method

1) Graphical method :

To solve a non-linear equation $f(x)=0$ we write $f(x) = g(x)-h(x)$ and we graph both functions g and h and the values of x at the intersection of the graphs of $g(x)$ and $h(x)$ are the solution of the equation $f(x)=0$ as it is shown in the following examples:

Example (1): By using the graphical method find the solution of the quadratic equation $2x^2-3x+1=0$.

Solution:

Let $f(x) = 2x^2-3x+1$

$\Rightarrow f(x) = 2x^2-(3x-1)$, then $g(x)=2x^2$ and $h(x) = 3x-1$

x	$g(x)$
-1	2
0	0
$\frac{1}{2}$	$\frac{1}{2}$
1	2

x	$h(x)$
-1	-4
0	-1
$\frac{1}{2}$	$\frac{1}{2}$
1	2