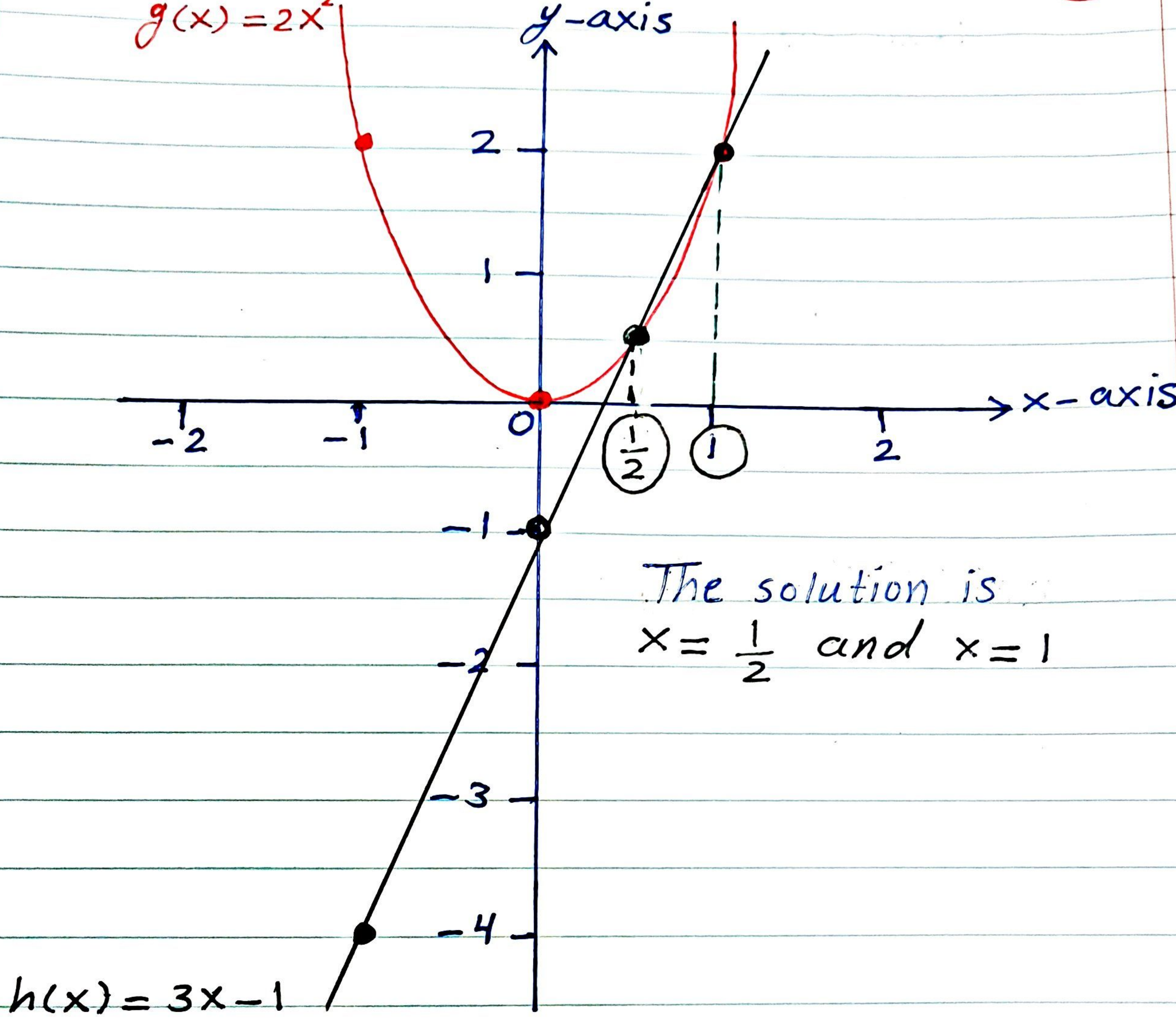


The solution is $x = \frac{1}{2}$ and $x = 1$

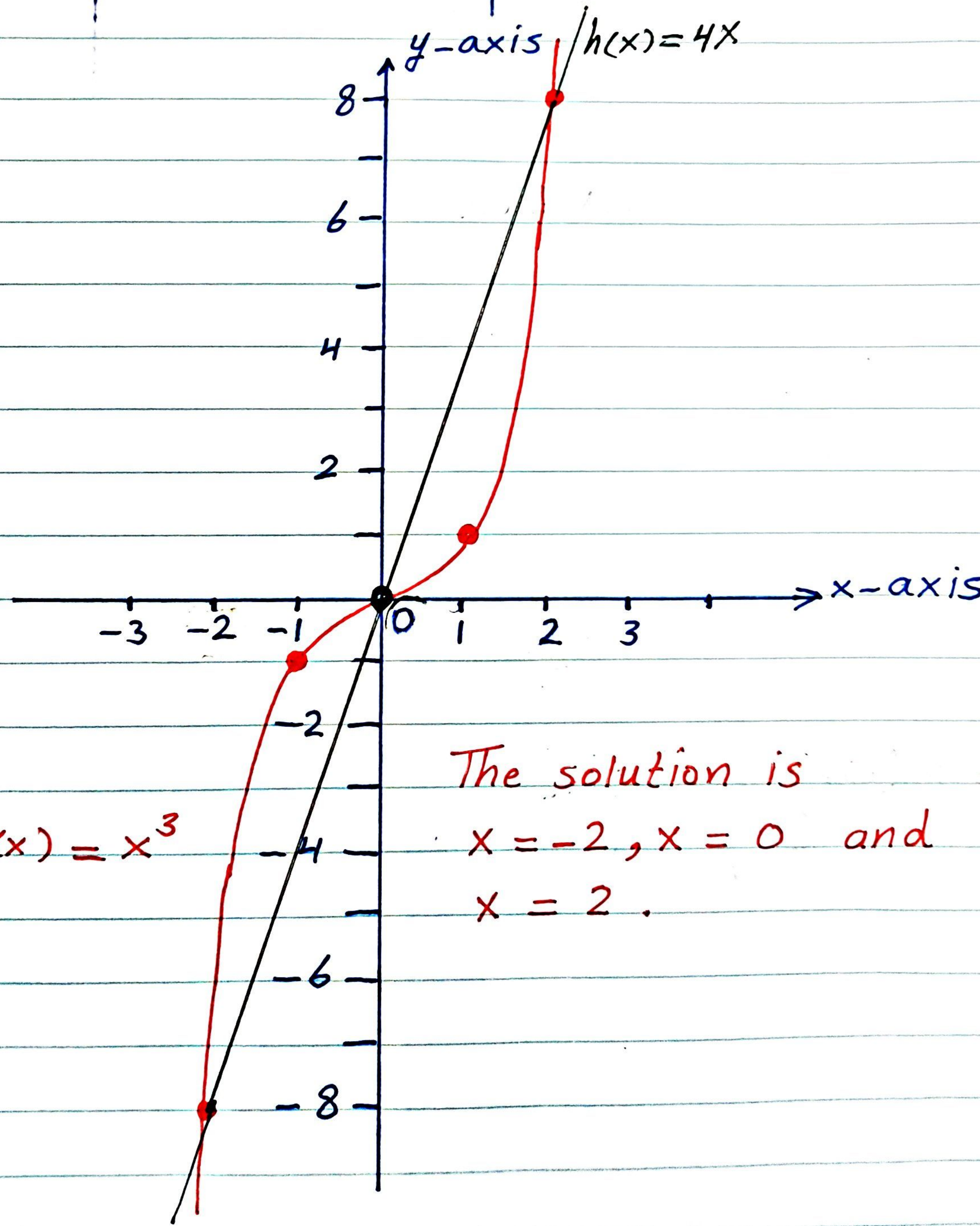
Example(2): By using the graphical method find the solution of the equation $x^3 - 4x = 0$.

Solution:

Let $f(x) = x^3 - 4x$, then $g(x) = x^3$ and $h(x) = 4x$

x	g(x)
-2	-8
-1	-1
0	0
1	1
2	8

x	h(x)
-2	-8
-1	-4
0	0
1	4
2	8



The solution is $x = -2$, $x = 0$ and $x = 2$.

2) Analytical method :

Analytical method based on the mean-value theorem. Let $f(x)$ be a real continuous function on the interval $[a, b]$, where a and b are real numbers such that $a < b$, if $f(a)$ and $f(b)$ have different signs, then there exists at least one real root on the interval $[a, b]$.

The accuracy to determine the location of roots depend on dividing the interval $[a, b]$ into subintervals.

The number of positive roots of $f(x)$ is the number of changes in signs of $f(x)$.

The number of negative roots of $f(x)$ is the number of changes in signs of $f(-x)$.

Example (1) : Find the number of positive and negative roots and their locations for the following equation by using the analytical method on the interval $[-1, 1]$:

$$f(x) = x^2 - x - 1$$

Solution :

$$f(x) = + x^2 - x - 1$$

$\therefore$ There is one positive root.

$$f(-x) = +(-x)^2 - (-x) - 1 = + x^2 + x - 1$$

∴ There is one negative root.

x	-1	0	1
$f(x)$	$+$	$-$	$-$

∴ There is one negative root in the subinterval $(-1,0)$, while there is no positive root in the interval $(-1,1)$.

Example (2): Find the number of positive and negative roots and their locations for the following equations by using the analytical method:

a) $f(x) = x^2 - x - 1$ in $I = [-1, 2]$.

b) $f(x) = x^3 - 5x^2 + 2x + 8$ in $I = [-4, 5]$.

Solution: a) From example 1 the function $f(x) = x^2 - x - 1$ has one positive root and one negative root.

x	-1	0	1	2
$f(x)$	$+$	$-$	$-$	$+$

∴ There is one negative root in the subinterval $(-1,0)$ and one positive root in the subinterval $(1,2)$.

b) $f(x) = +x^3 - 5x^2 + 2x + 8$