There are two positive roots.

$f(-x) = -x^3 - 5x^2 - 2x + 8$

There is one negative root.

x	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	$-$	$-$	$-$	0	$+$	$+$	0	$-$	0	$+$

$\therefore$ There is one negative root in the subinterval $(-2, 0)$ and two positive roots, one of them in the subinterval $(1, 3)$ and the other positive root in the subinterval $(3, 5)$.

3) Bisection method:

This method is used to find the root of an equation $f(x) = 0$ between a and b for a continuous function $f(x)$ when $f(a)$ and $f(b)$ are of different signs. Notice that if f is continuous between a and b and $f(a)$ and $f(b)$ are of different signs then the equation $f(x) = 0$ must have a root between a and b.

The method is as follows:

i) If $f(a) < 0$ and $f(b) > 0$ then the first approximation to the root is $x_1 = \frac{a+b}{2}$.
If $f(x_1) = 0$ then x_1 is a root of $f(x) = 0$.
Otherwise the root will be between a and x_1 if $f(x_1) > 0$, and the root will

be between x_1 and b if $f(x_1) < 0$.
The second approximation to the root is $x_2 = \frac{a+x_1}{2}$ if $f(x_1) > 0$ and $x_2 = \frac{x_1+b}{2}$ if $f(x_1) < 0$.
If $f(x_2) = 0$ then x_2 is a root of $f(x) = 0$.
Otherwise the root will be between
1) a and x_2 if $f(x_1) > 0$ and $f(x_2) > 0$
2) x_2 and x_1 if $f(x_1) > 0$ and $f(x_2) < 0$
3) x_2 and b if $f(x_1) < 0$ and $f(x_2) < 0$
4) x_1 and x_2 if $f(x_1) < 0$ and $f(x_2) > 0$.

Then the third approximation will be:

1) $x_3 = \frac{a+x_2}{2}$ if the root between a and x_2

2) $x_3 = \frac{x_2+x_1}{2}$ if the root between x_2 and x_1

3) $x_3 = \frac{x_2+b}{2}$ if the root between x_2 and b

4) $x_3 = \frac{x_1+x_2}{2}$ if the root between x_1 and x_2.

We continue the process untill the root is found.

ii) If $f(a) > 0$ and $f(b) < 0$ then the first approxiation to the root is $x_1 = \frac{a+b}{2}$.
If $f(x_1) = 0$ then x_1 is a root of $f(x) = 0$

Otherwise the root will be between a and x_1 if $f(x_1) < 0$, and the root will be between x_1 and b if $f(x_1) > 0$.

The second approximation to the root is

$$x_2 = \frac{a + x_1}{2} \text{ if } f(x_1) < 0 \text{ and } x_2 = \frac{x_1 + b}{2} \text{ if } f(x_1) > 0.$$

If $f(x_2) = 0$ then x_2 is a root of $f(x) = 0$.

Otherwise the root will be between

1) a and x_2 if $f(x_1) < 0$ and $f(x_2) < 0$
2) x_2 and x_1 if $f(x_1) < 0$ and $f(x_2) > 0$
3) x_2 and b if $f(x_1) > 0$ and $f(x_2) > 0$
4) x_1 and x_2 if $f(x_1) > 0$ and $f(x_2) < 0$.

Then the third approximation will be:

1) $x_3 = \frac{a + x_2}{2}$ if the root between a and x_2

2) $x_3 = \frac{x_2 + x_1}{2}$ if the root between x_2 and x_1

3) $x_3 = \frac{x_2 + b}{2}$ if the root between x_2 and b

4) $x_3 = \frac{x_1 + x_2}{2}$ if the root between x_1 and x_2.

We continue the process untill the root is found.

Example: By using the bisection method find an approximation root of the equation $x^3 - 4x - 7.6 = 0$ that lies between $a = 2$ and

$b = 3$. Carry out computations upto the third stage (or to two decimal places).

Solution: Let $f(x) = x^3 - 4x - 7.6$

$f(2) = 8 - 8 - 7.6 = -7.6$ (−ve)

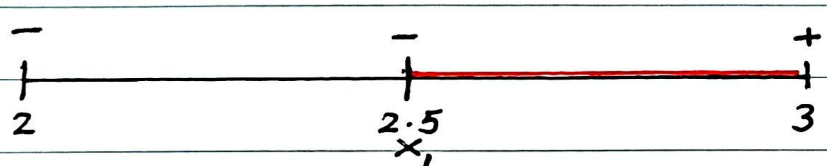
$f(3) = 27 - 12 - 7.6 = 7.4$ (+ve)

$\therefore$ a root lies between 2 and 3.

Thus the first approximation to the root is

$x_1 = \frac{2+3}{2} = 2.5$

and $f(x_1) = (2.5)^3 - 4(2.5) - 7.6 = -1.975$ (−ve)



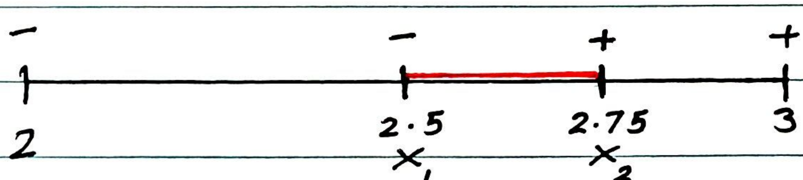
Hence the root lies between $x_1 = 2.5$ and 3.

Thus the second approximation to the root is

$x_2 = \frac{2.5+3}{2} = 2.75$

and $f(x_2) = (2.75)^3 - 4(2.75) - 7.6 = 2.196875$

(+ve)



Hence the root lies between $x_1 = 2.5$ and $x_2 = 2.75$.

Thus the third approximation to the root is

$x_3 = \frac{2.5 + 2.75}{2} = 2.625$

and $f(x_3) = (2.625)^3 - 4(2.625) - 7.6$

$= -0.012109375$

$\therefore$ The root $\bar{x}$ is approximately equal 2.625.